ESTUDIOS DE ECONOMÍA APLICADA VOL. 33-3 2015 PÁGS. 801 – 816

Appraising the Financial Sustainability of a Pension System with Signal Processing

PIERRE ROSTAN [a], RACHID BELHACHEMI [b], ALEXANDRA ROSTAN [a]

[a] *American University in Cairo, School of Business, AUC Avenue, P.O. Box 74 New Cairo 11835, Egypt. E-mail:* prostan@aucegypt.edu, arostan@aucegypt.edu

[b] *Xi'an Jiaotong-Liverpool University, Mathematical Sciences, 111 Ren Ai Road, Dushu lago Superior Educación Town, Suzhou Industrial Park, Suzhou, Jiangsu, 215123, China. E-mail:* rachid.belhachemi@xjtlu.edu.cn

ABSTRACT

One key issue of the Spanish pension system is its financial sustainability in regard to slumping fertility rate and rising longevity of the Spanish population. The paper presents a versatile and robust model that may help pension managers gain insight into the future Spanish pyramid of ages, from which they will appraise cash inflows and outflows of the pension system. The model forecasts ninety years of the Spanish population for each cohort of the pyramid of ages. Borrowed from the signal processing discipline, the model relies on the Burg method which fits a *p*th order autoregressive (AR) model to the input signal by minimizing (least squares) the forward and backward prediction errors while constrai-ning the AR parameters to satisfy the Levinson-Durbin recursión, then uses an infinite impulse response prediction error filter. Results add better perspective and insight to the Spanish population projection forecasted by the United Nations Population Division.

Keywords: Monte Carlo Simulation, Pyramid of Age, Pension System.

Evaluando la sostenibilidad financiera del sistema de pensiones con el procesamiento de la señal

RESUMEN

Una de las cuestiones clave del sistema de pensiones español es su sostenibilidad financiera en lo que respecta a caída tasa de fecundidad y el aumento de la longevidad de la población española. El artículo presenta un modelo versátil y robusto que puede ayudar a los administradores de pensiones ganan la penetración en el futuro pirámide española de los siglos, de la que evaluarán las entradas y salidas de efectivo del sistema de pensiones. El modelo pronostica noventa años de la población española para cada cohorte de la pirámide de edades. Tomado de la disciplina de procesamiento de señales, el modelo se basa en el método de Burg que se ajusta un modelo autorregresivo de orden *p* (AR) para la señal de entrada, reduciendo al mínimo (mínimos cuadrados) los errores de predicción hacia delante y hacia atrás mientras que restringir los parámetros AR para satisfacer las Levinson- Durbin, a continuación, se basa en un filtro de impulso infinito error de predicción de respuesta. Los resultados proporcionan una mejor perspectiva y una visión de la proyección de la población española prevista por la División de Población de las Naciones Unidas.

Palabras Clave: Simulación de Monte Carlo, Pirámide de edad, Sistema de Pensiones.

Classification JEL: C52, C63, J11, H55

Artículo recibido en mayo de 2015 y aceptado en mayo de 2015
Artículo disponible en versión electrónica en la página www.revista-eea.net, ref. ə-33304

ISSN 1697-5731 (online) – ISSN 1133-3197 (print)

1. INTRODUCTION

One key issue of the Spanish pension system is its financial sustainability in regard to slumping fertility rate and rising longevity of the Spanish population. 'The historical booms and busts in childbearing (Booth, 2006) have to a large degree provided the impetus for the more recent renewal of interest in demographic forecasting.' Baby bust and aging process of the population will bring a challenge to the pension system (Mora and Muñoz De Bustillo Llorente, 2004). In Spain, successive governments have sought for the optimal recipe to cope with population aging and its impact on pension system with additional constraints of economic downturn, political instability and social struggles. One approach to solve the problem has been to take example on the relative successes of reforms undertaken by other countries (Perez, 2006). In actuarial models focusing on projection of pension fund cash flows (Mettler, 2003), the derivation of the future expected cash flows may be broken down in four problems: '1) A population model describes the evolution of the insured in terms of their physical characteristics such as age and health status. 2) A salary model defines the future salary distribution of the active workforce. As contributions and some benefit components are calculated as percentage rates of the insured's salary, a salary model is a prerequisite for deriving the future cash flow profile. 3) A savings model outlines the accumulation process of contributions and interest proceeds. The future level of savings then serves as a basis for calculating the level of various benefit payments. 4) Finally, the cash flow model combines the results of the population model, the salary model and the savings model, thereby integrating the applicable contribution and benefit rates.' Therefore, the population variable is a key component of the model to appraise the pension fund inflows and outflows. This paper presents a versatile and robust model that may help pension managers gain insight into the future Spanish pyramid of ages, from which they will appraise cash inflows and outflows of the pension system. Borrowed from the signal processing discipline, the model forecasts ninety years of the Spanish population for each cohort of the pyramid of ages. Signal processing is a discipline which includes applications, algorithms, and implementations of processing signals. The principles of signal processing can be found in the classical numerical analysis techniques of the 17th century (Oppenheim and Schafer, 1975). Signal processing has applications in many fields such as electrical signals, audio signal processing, wireless communication, waveform generations, demodulation, filtering, equalization and seismology. Our paper adds 'population projection' to its application fields. In the methodology section, we present the reasons behind the choice of signal processing for forecasting population. The results section emphasizes the rationale of selecting signal processing for population projection. Section 2 reviews the literature concerning population forecasting. Section 3 presents the methodology in three steps. Section 4 presents the results and section 5 wraps up our findings.

2. LITTERATURE REVIEW

Besides national entities such as Instituto Nacional de Estadistica (Spanish Statistical Office) in Spain, a few international agencies produce population projections, namely the Population Division of the United Nations (U.N.), the World Bank and the United States Census Bureau. The latter defines population projections as 'estimates of the population for future dates. They are typically based on an estimated population consistent with the most recent decennial census and are produced using the cohort-component method'. The population projections of the U.S. Census Bureau are produced using a cohort-component method starting with an estimated base population. In the cohort-component method (U.S. Census Bureau, 2014 and Whelpton, 1936), 'the components of population change are projected separately for each birth cohort (persons born in a given year) based on past trends. For each year, for example, 2014 to 2060, the population is advanced one year of age using the projected age-specific survival rates and levels of net international migration for that year. A new birth cohort is added to the population by applying the projected age-specific fertility rates to the female population. Births, adjusted for infant mortality and net international migration, form the new population under one year of age. In its simplest form, the cohort component method is expressed as:

$$P_t = P_{t-1} + B_{t-1,t} - D_{t-1,t} + M_{t-1,t} \qquad (1)$$

where:

P_t = population at time *t*;

P_{t-1}= population at time *t-1*;

$B_{t-1,t}$ = births in the interval from time *t-1* to time *t*;

$D_{t-1,}$ = deaths in the interval from time *t-1* to time *t*; and

$M_{t-1,}$ = net migration in the interval from time *t-1* to time *t*'

This methodology follows a deterministic process, frequently used for population projections. Deterministic methods mean that they return a single projected value for each quantity of interest. Booth (2006) identifies three approaches to forecasting demographic processes such as mortality, fertility and migration: extrapolation, expectation (individual-level birth expectations or population-level opinions of experts), and theory-based structural modelling involving exogenous variables. The usual method of extrapolation is univariate ARIMA modelling (Box and Jenkins, 1976). Estimation is for example a survey of personal estimates of the chance of survival to a certain age that will help building life table estimates (Hauser and Willis, 2005). Structural models provide an explanation of demographic rates related to underlying socio-economic and proximate determinants. These models may integrate feedback mechanisms (Cohen, 1999 and Lee, 1990). For the past twenty-five years, authors have focused on forecasting demographic processes, mortality, fertility and migration and the three approaches cited above may have been intermingled: extrapolative methods are mixed with subjective expert opinion, exogenous variables with extrapolative models or structural models with extrapolation.

Developed in parallel with deterministic methods, probabilistic projections have gained the favor of population forecasters. Probabilistic projection methods provide a probability distribution of each quantity of interest, and therefore bring uncertainty to the projections. The uncertainty component has been increasingly recognized among authors. Before Lee and Tuljapurkar (1994), population projections used “high”, “medium” and “low” scenarios to indicate uncertainty and probability interpretations were rarely given. Their contribution was to make stochastic population forecasts by blending 'mathematical demography and statistical time series methods to estimate stochastic models of fertility and mortality and using the theory of random-matrix products to forecast various demographic measures and their associated probability intervals.' We can identify three broad approaches of probabilistic population projections: time series methods, expert-based and ex-post analysis. Time-series analysis methods (e.g. Tuljapurkar and Boe, 1999) use past time series of fore-cast inputs, such as fertility and mortality, to estimate a statistical time series model, which is then used to simulate a large number of random possible future trajectories. Lutz and al. (1998) promoted the expert based method where experts are required to provide distributions for each forecast input. Predictive distributions of forecast outputs will be extracted from the experts’ distributions with a stochastic method analogous to the time series method. Alho *et al.* (2008) are among proponents of ex-post analysis which is based on errors in past forecasts. Our benchmark, promoted and applied by the Population Division of the United Nations Secretariat uses the methodology presented in Raftery *et al.* (2012): it is based on Bayesian probabilistic population projections where 'the total fertility rate and female and male life expectancies at birth are projected probabilistically using Bayesian hierarchical models estimated via Markov chain Monte Carlo using United Nations population data for all countries. These are then converted to age-specific rates and combined with a cohort component projection model. This yields probabilistic projections of any population quantity of interest.'

Signal processing has not been applied to population projection in the past as we reviewed the literature about population projection. The method presented in section 3 may belong to the deterministic methods as opposed to probabilistic methods such as our benchmark.

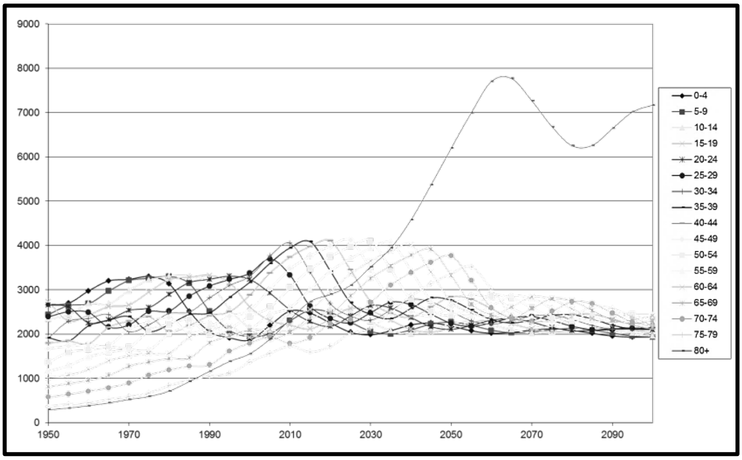
3. METHODOLOGY

The Spanish pyramid of ages is divided into seventeen classes of ages or cohorts: 0-4 years, 5-9, 10-14, 15-19, ..., 80+. We forecast the population values of the seventeen cohorts from 2015 to 2100. We benchmark our model to the Spanish population forecasts of the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat. The methodology is explained in three steps. Our intuitive approach of adapting signal processing to population projection comes from the observation of times series of cohorts constituting the Spanish pyramid of ages illustrated in Figure 1. The times series behave like signals. In communication systems, signal processing and electrical engineering, a signal refers to 'a function that conveys information about the behavior or attributes of some phenomenon' (Priemer, 1991). In electrical engineering, the embodiment of a signal in electrical form is made by a transducer that converts the signal from its original form to a waveform expressed as a current or a voltage, or an electromagnetic waveform, for example, an optical signal or radio transmission.

Figure 1

The 17 cohorts constituting the Spanish pyramid of ages for the 1950-2100 period. 1950-2010 data obtained by census; after 2010, population projection of the Population Division of the United Nations Secretariat

(in thousands)



Source: Own elaborations.

In Figure 1, we observe that the population time series oscillate in waveforms, more specifically propagate in sine waves. Overtime, some time series have their waves amplifying, some are amplifying then decaying, some are simply reducing in amplitude. In signal processing, wave propagation is how waves travel. Our model is able to capture the amplitude, the frequency, the trend of increasing or decreasing amplitude of the waves represented by the times series during their propagation through time. Based on the above observations which try to demonstrate the reasons why signal processing may be adapted to population projection, our research assumption is that population time series propagate through time like signals through space.

3.1. Step 1: interpolating data

For each of the seventeen cohorts of the Spanish pyramid of ages, we gather 13 historical population values from 1950 to 2010 with a 5-year step. To apply optimally our signal processing method to the fitting and forecasting problems, we need to generate more than 13 points: we get 600 observations for 60 years between 1950 and 2010 using cubic spline interpolation; we set the time interval *dt* to 0.10. We are thus able to apply signal processing to our database.

3.2. Step 2: fitting the time series of the Spanish population for the 1950-1985 period

Given x the times series of 600 observations per cohort, we generate a vector a of all-pole filter coefficients that model an input data sequence using the Levinson-Durbin algorithm (Levinson, 1946 and Durbin, 1960). We use the Burg method (1975) to fit a pth order autoregressive (AR) model to the input signal, x, by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion. x is assumed to be the output of an AR system driven by white noise. Vector a contains the normalized estimate of the AR system parameters, A(z), in descending powers of z:

$$\mathrm{H(z)}=\frac{\sqrt{\mathrm{e}}}{\mathrm{A(z)}}=\frac{\sqrt{\mathrm{e}}}{1+\mathrm{a}_2\mathrm{z}^{-1}+...+\mathrm{a}_{(\mathrm{p}+1)}\mathrm{z}^{-\mathrm{p}}} \tag{2}$$

Where $H(z)$ is the optimal linear predictor.

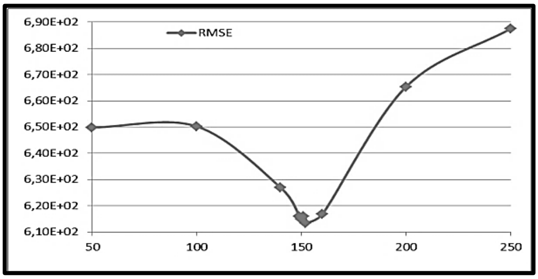
Since the method characterizes the input data using an all-pole model, the correct choice of the model order p is important. In order to determine the optimal pth order autoregressive coefficient for the database, we divide our initial sample of data of 13 population observations for each cohort in 8 and 5 observations, respectively from 1950 to 1985 and from 1990 to 2010. We re-apply the Burg method to fit a pth order autoregressive (AR) model to the input signal which is represented by 8 interpolated observations from 1950 to 1985 over 35 years, i.e. 350 observations after cubic spline interpolation. Then, we forecast the 5 population values for the years 1990, 1995, 2000, 2005 and 2010 over the 17 cohorts. We choose the model order p that minimizes the root mean square error (RMSE) defined in equation 3 between the observed values y_i and the values forecasted by the model $\hat{y}_i$ for the whole database of 5 observations x 17 cohorts.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{3}$$

Figure 2 illustrates the model order *p* versus the RMSE where *p*=152 minimizes the RMSE value. Part of these explanations is found in the signal processing section of the Matlab help file.

Figure 2

Model order *p* (X-axis) of the Burg method versus RMSE of the forecasting values over the 1990-2010 period



Source: Own elaborations.

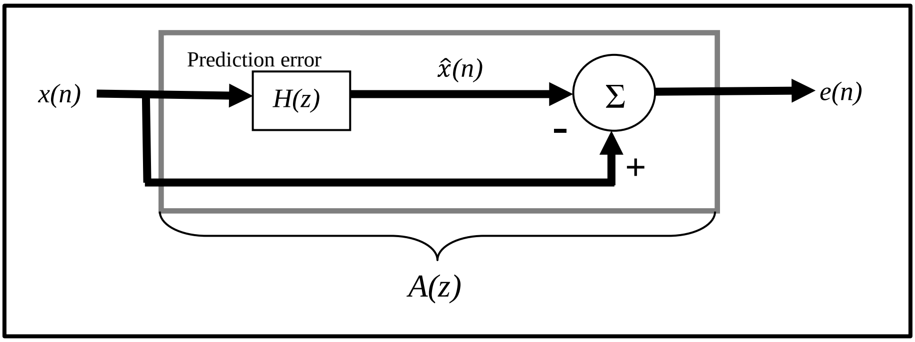
3.3. Step 3: forecasting the time series of the Spanish population for the 1990-2100 period

At step 2, we have seen that, in order to evaluate the model order *p*, we need to forecast a 5-year sample. The minimization of the forecasting error helps determining *p*. At step 3, we explain the forecasting method. We run the initial interpolated time series *x* from 1950 to 2010 (600 data per cohort) through the filter to get the filter state. In Figure 3, the prediction error, *e(n)*, can be viewed as the output of the prediction error filter *A(z)* shown below, where *H(z)* defined by equation 2 is the optimal linear predictor, *x(n)* is the input signal, and $\hat{x}(n)$ is the predicted signal.

Where *H(z)* and *A(z)* are defined in equation 2.

We finally use the IIR filter to extrapolate the Spanish population values per cohort from 2015 to 2100. Infinite impulse response (IIR) filters are digital filters with infinite impulse response. Unlike finite impulse response (FIR) filters, IIR filters have the feedback (a recursive part of a filter) and are known as recursive digital filters therefore.

Figure 3
Prediction error filter used for forecasting Spanish population over the 2015-2100 period



Source: Matlab, signal processing help file.

4. RESULTS

The Spanish pyramid of ages is divided in 17 cohorts: 0-4 years, 5-9, 10-14, 15-19, ..., 80+. We forecast the population values of the seventeen cohorts from 2015 to 2100 using the Burg method and an IIR filter normally applied to signal processing. We benchmark our model to the Spanish population projection of the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat. In section 4.1., we review some examples of time series obtained with signal processing. In section 4.2., we present results for the total population. In section 4.3., we attempt to explain the differences between population projections obtained with signal processing and the U.N. benchmark.

4.1. Examples of forecasted time series with signal processing

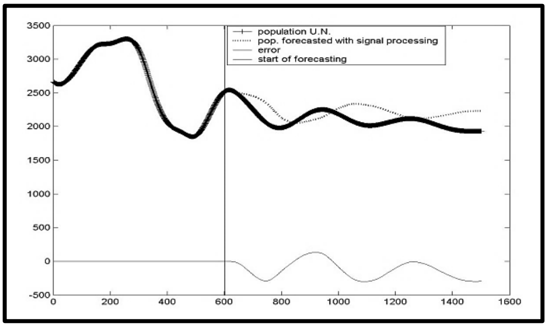
A first example is the cohort 0-4 represented by Figure 4.

The error (bottom line) measures the difference between the benchmark (population U.N.) and the population values forecasted with signal processing. First, we acknowledge the high fitting accuracy of signal processing using the Burg method coupled with an infinite impulse response prediction error filter, since as observed in Figures 4, 5 and 6, the error is zero for the in-sample section. Second, regarding the forecasted section, Figure 4 shows that the signal processing model captures with accuracy the decrease in amplitude of the signal wave form. The sinusoidal wave is countercyclical to the one forecasted by the U.N. Our model adjusts the frequency and amplitude of the forecasted sine wave to the information provided by past data based on signal processing theory. On the other hand, the U.N. benchmark is constructed with Bayesian probability assumptions, which explains the difference. As mentioned earlier, Bayesian probability models project probabilistically the total fertility rate and female and male life expectancies at birth using Bayesian hierarchical models estimated via Markov chain Monte Carlo. The projections are then converted to age-specific rates and combined with a cohort component projection model. Instead of simply extrapolating the time series like signal processing, Bayesian probability models derivate population cohorts' projections from core variables. This multistep methodology may suffer from a complexity factor.

Figure 4

Cohort 0-4 years; population forecasted with signal processing versus population projection of the United Nations Secretariat (population U.N.)

(in thousands)



Source: Own elaborations.

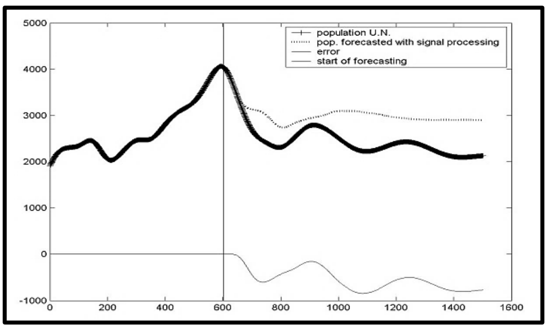
In addition, we observe in Figure 4 that the trend of the forecasted population composing the 0-4 years cohort points slightly downward with the benchmark whereas the signal processing model has a clear flat trend for the 90-year forecast, the latter being more optimistic about the baby bust.

A second example is the cohort 30-34 years represented by Figure 5.

Figure 5

Cohort 30-34 years; population forecasted with signal processing versus population projection of the United Nations Secretariat (population U.N.)

(in thousands)



Source: Own elaborations.

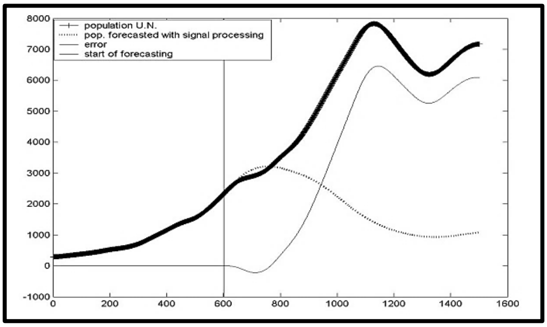
We observe that the error (bottom line) has a downtrend, meaning that the population forecast of the U.N. is again pessimistic compare to our model, expecting a continuous decline of the number of adults belonging to the cohort 30-34 years in Spain. In contrast, our model displays less fluctuations of the value of adults and is again anticipating, after the start of a sharp decline, stabilization until the end of the 21st century.

A third example is the cohort 80 years+ represented by Figure 6.

Figure 6

Cohort 80 years old and older; population forecasted with signal processing versus population projection of the United Nations Secretariat (population U.N.)

(in thousands)



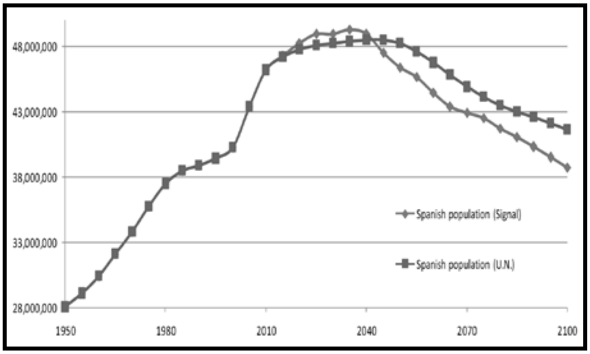
Source: Own elaborations.

This particular cohort of 80 years old and older displays a clear rupture between our model and the benchmark. Like all other cohorts, our model identifies a sine wave whose amplitude and frequency are explained by past data. On the contrary, the U.N. 80+ cohort benchmark points sharply upward for the remaining 90 years of the 21st century. The U.N. obviously assumes that medical progress and social protection of elderly people will make their cohort number skyrocketing, but do not take into consideration: 1)that epidemiologic factors such as new viruses, antibiotic resistant-bacterias, depletion of the immune system by genetically modified food, deterioration of the environment quality by global warming may slow down the rise of octogenarians; 2)that economic factors such as a deep and prolonged recession or an overflow of migrants may reduce the transfer payments of the Spanish government; 3)that political factors such as political instability, social struggles or war against terrorism may impair the social protection of the 80+ vulnerable population with government budget cuts. The U.N foresees the 80+ cohort jumping by more than 200% by the end of the century, which makes this figure rather unlikely. Our model is contrarian and forecasts a smooth decline of this cohort. The truth may be somewhere in the middle.

4.2. Results for the total population

Figure 7 illustrates the total Spanish population forecasted with signal processing and by the U.N.

Figure 7
Total population forecasted with signal processing versus population projection of the United Nations Secretariat (U.N.)



Source: Own elaborations.

The signal processing model forecasts higher values of the Spanish population from 2015 to 2040, then lower values from 2045 to 2100. The downtrend of the curve starting in 2045 is however parallel to the curve forecasted by the U.N., which makes our model overall consistent with the model applied by the U.N.

4.3. Explaining the differences between population projections obtained with signal processing and the U.N. benchmark

The forecasts of the total population obtained with the two methods converge: the Spanish population will decline steadily until the end of the century. However, the drivers of this decline are different depending on the method.

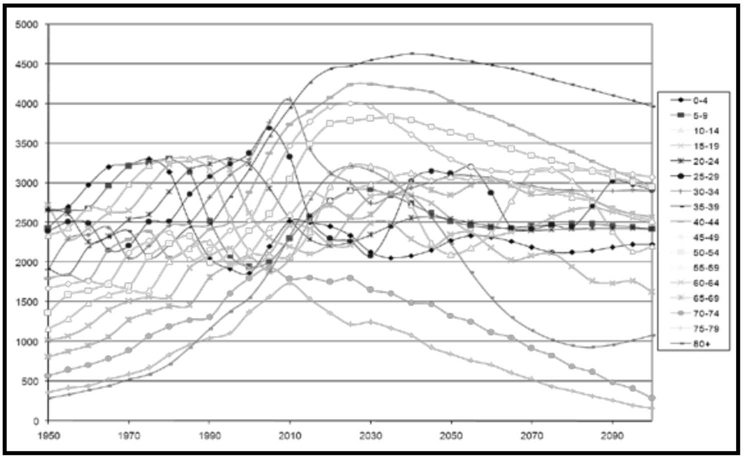
Figure 8 illustrates the 17 cohorts forecasted by signal processing which can be contrasted with the 17 cohorts forecasted by the U.N. represented in Figure 1. A general comment is that forecasted time series between 2015 and 2100 have the appearance of signal waves like the ones forecasted by the U.N. Peaks are followed by troughs like sine waves. However, time series forecasted by signal processing are not converging towards the 2.2 million-level as we observe in Figure 1 with U.N. projections. Figure 8 shows more diffuse times series, spreading out in 2100 between 4 million for the 35-39 cohort and 155,000 for the 75-79 cohort. In addition, we observe in Figure 8 that the cohorts of elderly people (60-64, 70-74, 75-79, 80+) decrease drastically in value at the end of the century compare to the 2015-levels. After 2040, all cohorts, except the

10-14, 25-29, 65-69 cohorts, have a clear downtrend, which explain the downtrend of the total population in Figure 7. The three exceptions will start their decline later.

Figure 8

The 17 cohorts constituting the Spanish pyramid of ages for the 1950-2100 period. 1950-2010 data obtained by census; after 2010, population projection obtained by signal processing

(in thousands)



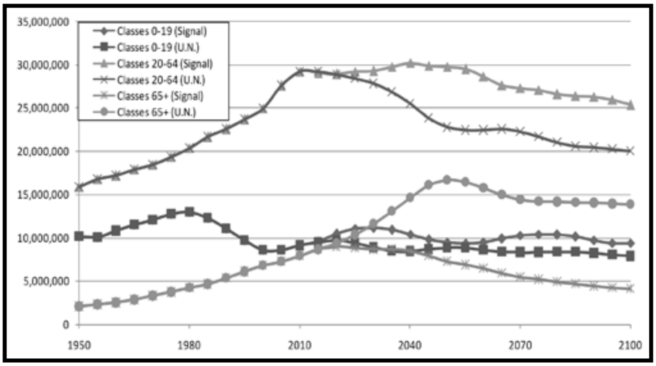
Source: Own elaborations.

We attempt to appraise the financial sustainability of the Spanish pension system by regrouping the 17 cohorts in three major cohorts, the 0-19, 20-64, 65+ as illustrated by Figure 9.

Starting our observation of Figure 9 with the upper curves, the working class 20-64 who feeds the pension system is at a lower level between 2015 and 2100 with the U.N. forecast (23.6 million on average) than the signal processing forecast (28.1 million on average). The 65+ cohort who depletes the pension system is at a significantly higher level between 2015 and 2100 with the U.N. forecast (13.6 million on average) than the signal processing forecast (6.6 million on average). Finally, the 0-19 cohort who is considered the next generation to feed the pension system is at a lower level between 2015 and 2100 with the U.N. forecast (8.6 million on average) than the signal processing forecast (10.0 million on average). The signal processing population projection is therefore more optimistic concerning financial sustainability of the Spanish pension system with 4.5 million more workers on average who feed the pension system over the period 2015-2100, 1.4 million more next-to-become-workers and 7 million less retirees who deplete the pension system.

Figure 9

Three combined cohorts, the 0-19, 20-64, 65+ constituting the Spanish pyramid of ages for the 1950-2100 period. 1950-2010 data obtained by census; after 2010, population projection obtained by signal processing versus U.N.

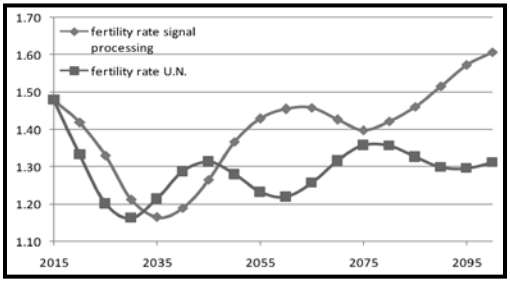


Source: Own elaborations.

Finally, focusing on the fertility rate defined as the ratio of live births to the population, expressed per 1000 population per year, we choose the cohort 0-4 as proxy of the live births that we adjust to the CIA World Factbook 2014 estimate for Spain, i.e. 1.48 per 1,000, assuming to be identical in 2015. We obtain Figure 10.

Figure 10

Forecasting the fertility rate by signal processing versus U.N. from 2015 to 2100



Source: Own elaborations.

In Figure 10, after reaching a low in 2035, the fertility rate forecasted by signal processing experiences a clear rebound in the subsequent years and reaches 1.6 at the end of the century versus 1.30 for the U.N. forecast. The signal processing model is again more optimistic in terms of sustainability of the Spanish pension system.

5. CONCLUSION

Our objective is to provide a versatile and robust model to pension managers, to gain insight into the future Spanish pyramid of ages from which they will appraise cash inflows and outflows of the pension system.

The model, based on signal processing, relies on the Burg method which fits a *p*th order autoregressive (AR) model to the input signal by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion, then uses an infinite impulse response prediction error filter, and assumes that population time series propagate through time like signals through space. The Spanish pyramid of ages is divided in 17 cohorts: 0-4 years, 5-9, 10-14, ..., 80+. We forecast the population of the 17 cohorts from 2015 to 2100. We benchmark our model to the projection of the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat, which bases their projection on a Bayesian probabilistic model.

We show that the overall population forecast of the two models is quiet similar, forecasting a downtrend of the population starting in 2045. However, the drivers of this decline are different depending on the method. The 80+ cohort of the U.N. follows a distinct pattern, a sharp uptrend for the last 90 years of the 21st century. The U.N. obviously assumes that medical progress and social protection of elderly people will make their cohort number skyrocketing. The U.N foresees the 80+ cohort jumping by more than 200% by the end of the century, which makes this figure rather unlikely. The signal processing model is contrarian and forecasts a smooth decline of this cohort. The truth may be somewhere in the middle. Regarding the 16 remaining cohorts, the U.N. model makes the cohorts following a sine pattern, which all converge towards a 2.2 million-level at the end of the century. Time series forecasted by signal processing are not converging towards a 2.2 million-level; they are more diffuse, but have a clear downtrend in motion after 2040.

In an attempt to appraise the financial sustainability of the Spanish pension system, we regroup the 17 cohorts in three major cohorts, the 0-19, 20-64, 65+. We show that over the period 2015-2100, the population projection by signal processing is more optimistic than the U.N. in regard to the financial sustainability of the Spanish pension system with an average 4.5 million more workers than the U.N., workers that add contributions to the pension system, 1.4 million more next-to-become-workers and 7 million less retirees, who deplete the pension system. Finally, focusing on fertility rate, we show that, after reaching a low in 2035, the fertility rate forecasted by signal processing experiences a clear rebound in the subsequent years and reaches 1.6 at the end of the century versus 1.30 for the U.N. forecast. The signal processing model offers again a better outlook of the Spanish pension system.

The signal processing model should therefore bring to pension managers a clearer picture of population projection for a better appraisal of the financial sustainability of the Spanish pension system.

BIBLIOGRAPHY REFERENCES

ALHO, J.M., JENSEN S.E.H. and LASSILA, J. (2008). *Uncertain Demographics and Fiscal Sustainability*. Cambridge: Cambridge University Press.

BOOTH, H. (2006). “Demographic forecasting: 1980 to 2005 in review“. International Journal of Forecasting, 22, pp. 547-581.

BOX, G. E. P. and JENKINS, G. M. (1976). *Time Series Analysis: Forecasting and Control. Revised Edition*, San Francisco: Holden Day.

BURG, J.P. (1975). “Maximum Entropy Spectral Analysis“. Available at: http://sepwww.stanford.edu/theses/sep06/ [May 12, 2015].

COHEN, J. E. (1999). “Should population projections consider "limiting factors" - and if so how?“. In LUTZ, W., VAUPEL, J. W. and AHLBURG, D. A. (Eds.). *Frontiers of Population Forecasting.* New York: Population Council. A supplement to Population and Development Review, 24 (1998), pp. 118-138.

DURBIN, J. (1960). “The fitting of time series models“. Revue de l'Institut International de Statistique, 28, pp. 233-44.

HAUSER, R. M. and WILLIS, R. J. (2005). “Survey design and methodology in the Health and Retirement Study and the Wisconsin Longitudinal Study“. In WAITE, L. J. (Ed.). *Aging, Health, and Public Policy: Demographic and Economic Perspectives*, *New York: Population Council*, Supplement to Population and Development Review, 30 (2004).

LEE, R. D. (1990). “Long-run global population forecasts: a critical appraisal“. *Population and Development Review,* 16, pp. 44-71.

LEE, R.D. and TULJAPURKAR, S. (1994). “Stochastic population forecasts for the United States: Beyond high, medium, and low“. *Journal of the American Statistical Association,* 89(428), pp.1175-1189.

LEVINSON, N. (1946). “The Wiener RMS (root mean square) error criterion in filter design and prediction“. *Journal* of *Mathematical Physics*, 25, pp. 261-78.

LUTZ, W., SANDERSON W.C. and SCHERBOV S. (1998). “Expert-based probabilistic population projections“. *Population and Development Review*, 24, pp. 139-155.

METTLER, U. (2003). “Projecting Pension Fund Cash Flows“, ASTIN colloquia, IAA (International Actuarial Association), Zurich, Switzerland. www.actuaries.org/ASTIN/Colloquia/Zurich/Mettler.pdf (Accessed on May 24, 2015).

MORA, F.E., MUÑOZ DE BUSTILLO LLORENTE, R. (2004). “Mitos y falacias populares en el debate acerca de los sistemas de pensiones“. *Estudios de economía aplicada*, 22(2), pp. 289-316.

OPPENHEIM, A. V. and SCHAFER, R. W. (1975). *Digital Signal Processing*, Upper Saddle River. New Jersey: Prentice Hall.

PEREZ, J.I.A. (2006). “La reforma de los sistemas de pensiones en Europa del Este y su impacto sobre la eficiencia y la equidad“. *Estudios de economía aplicada*, 24(2), pp. 783-814.

PRIEMER, R. (1991). *Introductory Signal Processing.* Singapore: World Scientific.

RAFTERY, A.E., LIB, N., ŠEVČÍKOVÁC, H., GERLANDB, P. and HEILIGB, G.K. (2012). “Bayesian probabilistic population projections for all countries“. *Proceedings of the National Academy of Sciences*, 109(35), pp. 13915-13921. doi:10.1073/pnas. 1211452109. Available at: http://www.pnas.org/content/109/35/13915.full.pdf+html [Accessed on May 12, 2015].

TULJAPURKAR, S. and BOE, C (1999). “Validation, probability-weighted priors, and information in stochastic forecasts”. *International Journal of Forecasting*, 15, pp. 259-271.

U.S. Census Bureau (2014). “Methodology, Assumptions, and Inputs for the 2014 National Projections“. Available at: http://www.census.gov/population/projections/files/methodology/methodstatement14.pdf [Accessed on May 12, 2015].

WHELPTON, P.K. (1936). “An empirical method for calculating future population”. *Journal of the American Statistical Association*, 31, pp. 457-473.