Estudios de Economía Aplicada Vol. 25-1, 2007. Págs. 341- 364

Adjustment of the Standard WACC Method to Subsidized Loans: A Clarification

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ABSTRACT

In this article, we show how to value projects financed by subsidized loans using the standard WACC method, with three distinct assumptions concerning the debt ratio targeted by the firm. In fact, the subsidized loan amount used to calculate this debt ratio can be determined according to book value, economic value or market value. This three definitions are equivalent when considering a non-subsidized loan. In each case, the value of a subsidized loan is determined with the help of a general dynamic non-linear model for the selection of projects with the option of subsidized financing. As a result, when considering economic value, we find the adjustment advocated by Myers (1974) in his Adjusted Present Value approach.

Keywords: WACC, subsidized loan, project valuation, capital budgeting.

Ajuste del metodo estandar del Costo medio ponderado del Capital (CMPC) en préstamos subsidiados: Una Aclaracion.

RESUMEN

Este articulo describe la manera de valorar proyectos financiados con préstamos subsidiados usando el método estandar del costo medio ponderado del capital, bajo tres supuestos concernientes al nivel de endeudamiento fijado por la empresa. El valor de los préstamos subsidiados usado para calcular el nivel de deuda puede ser determinado segun el valor contable, valor economico o valor de mercado. Estas tres definiciones son equivalentes cuando se consideran los prestamos sin subsidios. En cada caso, el valor de un prestamo subsidiado es determinado con la ayuda de un modelo general dinamico no-linear para la seleccion de proyectos con la opcion de financiacion subsidiada. Como resultado, cuando se considera un valor economico, se consigue la modificacion sugerida por Myers (1974) en su metodo del valor presente ajustado.

Palabras Clave: CMPC, prestamo subsidiado, estudios de rentabilidad.

Clasificación JEL: G30, G31, G32.

Articulo recibido en Mayo de 2005 y aceptado para su publicación en Diciembre de 2006.

Artículo disponible en versión electrónica en la página www.revista-eea.net, ref.: e-25109.

ISSN 1697-5731 (online) – ISSN 1133-3197 (print)

1. INTRODUCTION

The results presented in this article are applicable to the valuation of projects financed by subsidized loans using the standard WACC method, with different assumptions concerning the target debt ratio (or financial leverage) to be satisfied on a company-wide basis. A subsidized loan is a loan granted to a company at an interest rate lower than the market rate. For instance, international or governmental agencies offer subsidized financing as an incentive to undertake certain projects.

In the standard WACC method, a project's net present value is calculated by discounting the after-tax operating flows of the project at a rate reflecting both equity and debt required rates of return (i.e., both cost of equity and interest rate on debt). This discount rate is defined as the firm's after-tax weighted average cost of capital (see for instance Boudreaux and Long (1979)). In practice, the assumption is that the cost of equity, the after-tax cost of debt and the debt ratio (debt value divided by the sum of equity and debt values) are constant across time. A key point to note about this method is that, in the absence of special forms of financing (such as loan subsidies), only operating flows are taken into account when calculating the cash flows. The effects of financing flows, rather than being treated explicitly, are handled implicitly via the discount rate. As emphasized by Chambers et al. (1982), the standard WACC method is "well known and widely used in industry".

The literature proposes a valuation formula for subsidized financing that is valid only when the Adjusted Present Value is used (Brealey and Myers, 2003). This formula, which we will call "E-adjustment", consists of the sum of the after-tax interest expense differentials discounted at the after-tax cost of the non-subsidized loan (or, equivalently, the amount of the subsidized loan less the value of the after-tax cash flows related to that loan discounted at the after-tax cost of the non-subsidized loan). Its derivation, which considers the optimal capital structure specified exogenously, is not based on the assumption of maintaining a target debt ratio. To our knowledge, the only result proposed using the standard WACC method is that advanced, but not formally proved, by Babusiaux (1990) who suggests an adjustment (termed here the "B-adjustment") equal to a sum of after-tax interest expense differentials discounted at the after-tax weighted average cost of capital of the firm. This adjustment is mentioned by Jennergren (2005) as "a special case of a more general procedure that is often used in actual practice, to discount the differences in cash flows between two alternatives at the company's discount rate, i.e., the company's WACC". More recently, Pierru (2006) proposes to maximize the firm's adjusted present value, while maintaining a constant debt ratio, but he only considers two-period projects and a unique definition of the debt ratio. Because of this lack of generality, his result has little practical interest.

We prove in this article that the value of a subsidized loan depends on the definition of the debt ratio - or the financial leverage (debt value divided by equity value)

- that must be satisfied on a firm-wide basis. We will consider that these ratios are calculated with respect to the economic value of the equity. The firm's debt, on the other hand, can be defined in three different ways:

- in *book value* (present value of pre-tax cash flows discounted at the pre-tax cost of the subsidized loan),

- in *economic value* (present value of after-tax cash flows discounted at the after-tax cost of the non-subsidized loan),

- in *market value* (present value of the pre-tax cash flows discounted at the pre-tax cost of the non-subsidized loan).

These three definitions, of course, result in the same value for a non-subsidized loan. For each definition of the debt ratio, the value of a subsidized loan is determined with the help of a general dynamic model for projects selection with the option of subsidized financing.

Consistently with the normative theory of finance which assumes that the objective of corporate managers is to maximize the value of the firm to existing shareholders, the objective of the models developed here is to maximize the value of equity, subject to the constraint of a constant debt ratio with debt being expressed either in book value (first model), in economic value (second model) or in market value (third model). For each of the models, the dual variables associated with the various constraints are determined at the optimal financing allocation. They enable a marginal analysis and are therefore useful for decision-making purposes.

The transition from marginal analysis to the calculation of the firm's value is made by observing that, once the project sizes have been established, the optimization programs become linear programs, as the projects' operating cash flows and maximum loan amounts can then be considered as right-hand-side coefficients of the equations. By using the property of strong duality, the previously determined dual variables can then be used to determine the firm's value.

The three approaches lead to the same result as regards the valuation of operating cash flows for the projects. These cash flows must be discounted to present value at the firm's after-tax weighted average cost of capital, defined as the average of the cost of equity and of the marginal after-tax cost of debt (after-tax non-subsidized interest rate). On the other hand, the first model entails a B-adjustment for the valuation of a subsidized loan, while the second model entails an E-adjustment. When the debt ratio concerns debt defined at market value, the adjustment (termed the "M-adjustment") used to calculate the value of the subsidized loan proves more difficult to handle. For a given financing strategy (i.e., once loans are allocated), the equity residual method, the standard WACC method with a B-adjustment, an E-adjustment and an M-adjustment naturally result in the same firm's value. We provide a numerical example to illustrate this consistency.

Section 2 introduces the notation and assumptions used in this article. Book, economic and market values of a subsidized loan are explicitly defined. The corresponding valuation formulas (cash flow adjustments) are successively derived in sections 3, 4 and 5. Their consistency is studied in section 6, which provides a numerical illustration. Conclusions are offered in section 7.

2. NOTATION AND DEFINITIONS

Notation and general assumptions

The following notation has been adopted:

x_u dimension of the project P_u

$F_{u,n}(x_u)$ after-tax operating cash flow produced in year n by the project P_u

$V_{u,n}$ economic value of the project P_u at the end of year n, determined according to the equity residual method

r interest rate of a non-subsidized loan (marginal cost of debt)

$B_{u,n}$ non-subsidized loan amount allocated to the project P_u at the end of year n

r_u interest rate of the subsidized loan to which the project P_u has access

$S_{u,n}$ book value at the end of year n of the subsidized loan allocated to the project P_u

$S^l_{u,n}(x_u)$ maximum value that can be reached by $S_{u,n}$

$Y_{u,n}$ economic value at the end of year n of the subsidized loan allocated to the project P_u

$W_{u,n}$ market value at the end of year n of the subsidized loan allocated to the project P_u

θ income tax rate to which the firm's earnings are subject

w debt ratio set by the firm

c the firm's cost of equity

T horizon (in years) of the model

The value of each project is calculated according to the equity residual method (see for instance Chambers et al. (1982)), by imputing to the project all cash flows related to the loans that are associated with it. The objective is to maximize the net present value of the firm's equity at year 0. Over the period in question of T years, the firm has z investment opportunities or projects that can be undertaken. These projects constitute continuous ranges of investment independent from each other.

$F_{u,n}$ denotes the (after-tax) operating cash flow produced in year *n* ($n \in \{0,...,T\}$) by the project P_u ($u \in \{1,...,z\}$). This cash flow is assumed to be a continuous function $F_{u,n}(x_u)$, differentiable with respect to the size x_u of the project P_u. Given these assumptions, to define bounded project sizes, simply consider the functions $F_{u,n}(x_u)$ so that, in the neighborhood of the bound, the investment cost grows very rapidly (or revenues fall sharply). Certain projects can be undertaken beginning in year 0, and others in later years. For a project P_q starting in year *t*, one should then take $F_{q,k}(x_q) = 0$ for $k < t$. The value of a project P_u in *T* is given as a function of x_u and will be notated $V_{u,T}(x_u)$.

The maximum amount of the subsidized loan at the rate r_u that can be allocated in year *n* to the project P_u, denoted $S^l_{u,n}(x_u)$, is also assumed to be a continuous function and differentiable with respect to x_u. The definition of $S^l_{u,n}(x_u)$ will be specific to each project (for certain projects, for instance, it may be equal to a percentage of investment expenditure). A certain number of projects may not be eligible for subsidized financing. In that case, for each of these projects, one should simply set out: $S^l_{u,n}(x_u) = 0, \ \forall x_u$.

We also hypothesize that it is possible to allocate to each project a loan contracted at the non-subsidized interest rate *r* (with $r_u \leq r, \forall u$). The rate *r* here corresponds to a marginal cost of the loan. In principle, we could just as well not have introduced it into the model. At the optimum debt allocation process, the marginal cost of the loan would then be equal to the highest interest rate r_u at which a loan is contracted. Introducing *r* at the start allows one to deal with the most general case where a project is partly financed by subsidized financing and partly by non-subsidized financing. The rate *r* is assumed to be constant over the horizon *T* of the model.

The firm undertakes to satisfy a target debt ratio *w* each year (or, equivalently, financial leverage of $\frac{w}{1-w}$) for all of its projects (all assumed to belong to the same risk class). Its corresponding cost of equity is *c*.

The economic function to be maximized is equal to the sum of the equity residual net present values, at $n = 0$, of the entire set of projects undertaken. The variables x_u, as well as the loan amounts $S_{u,n}$ and $B_{u,n}$ to be allocated to the various projects each year, represent the program's control variables. These variables are subject to non-negativity constraints, which is not the case for variables $V_{u,n}$ (since the value of a project can be negative during certain years).

The equity value $V_{u,n}$ of a project P_u given at the end of year *n* is:

$$V_{u,n} = \sum_{k=n+1}^{T} \frac{F_{u,k}\left(x_u\right) + B_{u,k} - \left(1+\left(1-\theta\right)r\right)B_{u,k-1} + S_{u,k} - \left(1+\left(1-\theta\right)r_u\right)S_{u,k-1}}{\left(1+c\right)^{k-n}} + \frac{V_{u,T}}{\left(1+c\right)^{T-n}}$$

$V_{u,n}$ and $V_{u,n+1}$ are therefore related by the following equation:

$$V_{u,n} = \frac{V_{u,n+1} + F_{u,n+1}\left(x_u\right) + B_{u,n+1} - \left(1+\left(1-\theta\right)r\right)B_{u,n} + S_{u,n+1} - \left(1+\left(1-\theta\right)r_u\right)S_{u,n}}{1+c}$$

Alternative definitions of the amount of subsidized loan attached to a project

The amount of debt used to calculate the firm's debt ratio can be expressed either in book value, in economic value, or in market value. A preliminary definition of each of these three values is given in the introduction (section 1). Let us make them clear for a given subsidized loan.

In year *n*, the book value of the subsidized loan attached to the project P_u is $S_{u,n}$, as by definition we have:

$$S_{u,n} = \sum_{k=n+1}^{T} \frac{\left(1+r_u\right)S_{u,k-1} - S_{u,k}}{\left(1+r_u\right)^{k-n}}$$

In year *n*, the economic value $Y_{u,n}$ of this subsidized loan is defined as follows:

$$Y_{u,n} = \sum_{k=n+1}^{T} \frac{\left(1+\left(1-\theta\right)r_u\right)S_{u,k-1} - S_{u,k}}{\left(1+\left(1-\theta\right)r\right)^{k-n}}$$

Which can be written:

$$Y_{u,n} = \frac{Y_{u,n+1} + \left(1+\left(1-\theta\right)r_u\right)S_{u,n} - S_{u,n+1}}{1+\left(1-\theta\right)r}$$

In year *n*, the market value $W_{u,n}$ of this subsidized loan is defined as follows:

$$W_{u,n} = \sum_{k=n+1}^{T} \frac{\left(1+r_u\right)S_{u,k-1} - S_{u,k}}{\left(1+r\right)^{k-n}}$$

Or, equivalently:

$$W_{u,n} = \frac{W_{u,n+1} + (1+r_u)S_{u,n} - S_{u,n+1}}{1+r}$$

3. MODEL WITH A DEBT RATIO CONSIDERING THE BOOK VALUE OF DEBT

The program is written as follows:

$$Max \sum_{u=1}^{z} \left(V_{u,0} + F_{u,0}(x_u) + B_{u,0} + S_{u,0}\right)$$

$$s.t. \begin{cases} (u=1,...,z)\ (n=0,...,T-1)\ \ S_{u,n} - S^l_{u,n}(x_u) \leq 0 \quad (1) \\ (u=1,...,z)\ (n=0,...,T-2)\ \ (1+c)V_{u,n} - V_{u,n+1} - B_{u,n+1} - S_{u,n+1} \\ \qquad + \left(1+(1-\theta)r\right)B_{u,n} + \left(1+(1-\theta)r_u\right)S_{u,n} - F_{u,n+1}(x_u) = 0 \quad (2) \\ (u=1,...,z)\ \ (1+c)V_{u,T-1} - V_{u,T}(x_u) - \left(1+(1-\theta)r\right)B_{u,T-1} - \left(1+(1-\theta)r_u\right)S_{u,T-1} - F_{u,T}(x_u) = 0 \quad (3) \\ (n=0,...,T-1)\ \ (1-w)\sum_{u=1}^{z}\left(B_{u,n} + S_{u,n}\right) - w\sum_{u=1}^{z} V_{u,n} = 0 \quad (4) \\ (u=1,...,z)\ (n=0,...,T-1)\ \ B_{u,n} \geq 0 \quad (5) \\ (u=1,...,z)\ (n=0,...,T-1)\ \ S_{u,n} \geq 0 \quad (6) \\ (u=1,...,z)\ \ x_u \geq 0 \end{cases}$$

We will use Kuhn-Tucker's conditions to determine the values of the dual variables. The dual variables associated with constraints of type (1), (2), (3) and (4) will be respectively notated $\lambda_{u,n}$, $\pi_{u,n}$, $\pi_{u,T-1}$ and μ_n. The dual relations are then written:

$$\begin{cases} (u=1,...,z)\ (n=0,...,T-1)\ \ \lambda_{u,n} \geq 0 \\ (u=1,...,z)\ \ 1-\pi_{u,0}(1+c)+w\mu_0 = 0 \quad (7) \\ (u=1,...,z)\ \ 1-\lambda_{u,0}-\pi_{u,0}\left(1+(1-\theta)r_u\right)-(1-w)\mu_0 \leq 0 \quad (8) \\ (u=1,...,z)\ \ 1-\pi_{u,0}\left(1+(1-\theta)r\right)-(1-w)\mu_0 \leq 0 \quad (9) \\ (u=1,...,z)\ (n=1,...,T-1)\ \ -\pi_{u,n}(1+c)+\pi_{u,n-1}+w\mu_n = 0 \quad (10) \\ (u=1,...,z)\ (n=1,...,T-1)\ \ -\lambda_{u,n}-\pi_{u,n}\left(1+(1-\theta)r_u\right)+\pi_{u,n-1}-(1-w)\mu_n \leq 0 \quad (11) \\ (u=1,...,z)\ (n=1,...,T-1)\ \ -\pi_{u,n}\left(1+(1-\theta)r\right)+\pi_{u,n-1}-(1-w)\mu_n \leq 0 \quad (12) \\ (u=1,...,z)\ \ \dfrac{dF_{u,0}(x_u)}{dx_u}+\sum_{n=0}^{T-1}\left(\pi_{u,n}\dfrac{dF_{u,n+1}(x_u)}{dx_u}+\lambda_{u,n}\dfrac{dS^l_{u,n}(x_u)}{dx_u}\right)+\pi_{u,T-1}\dfrac{dV_{u,T}(x_u)}{dx_u} \leq 0 \quad (13) \end{cases}$$

In year 0 (as in other years), the firm first considers the possibility of a subsidized loan with the lowest interest rate. The corresponding type (1) constraint is then binding. It then considers other possibilities for subsidized financing in ascending interest rate order. We will assume[1] that all type (1) constraints are binding. Loans are therefore contracted at rate *r*. There is thus at least one project *v* for which $B_{v,0} > 0$. For this project, the corresponding constraint (5) is not binding; the inequality (9) is therefore an equality which, associated with the equation (7), gives the following system:

$$1-\pi_{v,0}\left(1+(1-\theta)r\right)-(1-w)\mu_0 = 0$$
$$1-\pi_{v,0}(1+c)+w\mu_0 = 0$$

It thus follows that: $\pi_{v,0} = \dfrac{1}{1+(1-w)c+w(1-\theta)r}$ and

1 If this were not the case, the interest rate of the last loan contracted would assume the role of r in all subsequently derived results. This amounts to changing the value of the marginal rate of the debt of the firm.

$$\mu_0 = \frac{c-(1-\theta)r}{1+(1-w)c+w(1-\theta)r}$$

According to equation (7), we have:

$$u=1,....,z \qquad \pi_{u,0} = \pi_{v,0} = \frac{1}{1+(1-w)c+w(1-\theta)r}$$

Projects eligible for subsidized financing in year 0 have a non-binding type (6) constraint: $S_{u,0} > 0$. The inequality (8) is therefore an equality:

$$1-\lambda_{u,0} - \pi_{u,0}\left(1+(1-\theta)r_u\right) - (1-w)\mu_0 = 0$$

We obtain: $\lambda_{u,0} = \dfrac{(1-\theta)(r-r_u)}{1+(1-w)c+w(1-\theta)r}$

The interpretation of these dual variables does not pose any particular problem. In the first place, we would remark that the variables $\pi_{u,0}$ represent the discount factor that must be used with the standard WACC method. Notating k_w the discount rate from year 0 to year 1, we have:

$$1+k_w = \frac{1}{\pi_{u,0}} = 1+(1-w)c+w(1-\theta)r$$

Or: $k_w = (1-w)c + w(1-\theta)r$

k_w can be described as an after-tax weighted average cost of capital (weighted average of the cost of equity and of the marginal after-tax cost of debt). An increase in the right-hand side of constraint (2) implicitly corresponds to an increase in the after-tax operating cash flow. The value of the objective function increases by an amount equal to the increase in operating cash flow discounted at the weighted average cost of capital (as is the case with the standard WACC method).

Each of the dual variables $\lambda_{u,0}$, associated with a type (1) constraint, represents the increase in value of the objective function if the firm has the option of borrowing one additional euro at the subsidized interest rate r_u. In order to maintain the debt ratio *w*, one euro less is borrowed at rate *r* (the highest rate). The gain for the shareholder is then equal to the interest expense differential (calculated after tax) discounted at the after-tax weighted average cost of capital of the firm. This result is in agreement with the proposals advanced but not proved by Babusiaux (1990) and reformulations by Babusiaux and Pierru (2001). We will come back to this point later.

We will now generalize these results to the following years. For any project P_u, we rewrite in year n the equations (10) and (12):

$$-\pi_{u,n}(1+c)+\pi_{u,n-1}+w\mu_n=0$$

$$-\pi_{u,n}(1+(1-\theta)r)+\pi_{u,n-1}-(1-w)\mu_n\leq 0$$

Consider any two projects v and w. The equation (10) proves that if $\pi_{v,n-1}=\pi_{w,n-1}$ then $\pi_{v,n}=\pi_{w,n}$. Since this assumption was verified for year 0, there is in fact equality of these dual variables each year. We will once again assume that at least one loan is contracted at rate r in year n ($\exists\ u/B_{u,n}>0$). For the project u to which this loan is allocated, the inequality (12) is an equality.

We obtain: $\dfrac{\pi_{u,n}}{\pi_{u,n-1}}=\dfrac{1}{1+(1-w)c+w(1-\theta)r}$

And finally, for any project:

$$\pi_{u,n}=\frac{1}{\left(1+(1-w)c+w(1-\theta)r\right)^{n+1}}=\frac{1}{(1+k_w)^{n+1}}$$

Moreover, projects benefiting from a loan $S_{u,n}$ in year n have their constraint (1) binding. The equation (11) then corresponds to an equality, which allows us to write:

$$\lambda_{u,n}=\pi_{u,n}\left((1-\theta)r-(1-\theta_u)r_u\right)=\frac{(1-\theta)r-(1-\theta_u)r_u}{\left(1+(1-w)c+w(1-\theta)r\right)^{n+1}}$$

Or: $\lambda_{u,n}=\dfrac{(1-\theta)r-(1-\theta_u)r_u}{(1+k_w)^{n+1}}$

The ability in year n to contract an additional euro at the subsidized rate r_u makes it possible to increase the sum of the net present values by an amount equal to the discounted after-tax interest expense differential. The interpretation is the same as stated above: the additional loan amount contracted at rate r_u is substituted for an equivalent loan amount contracted at rate r.

We can now replace the dual variables by their respective values in the equation (13):

$$\frac{dF_{u,0}(x_u)}{dx_u}+\sum_{n=0}^{T-1}\frac{\frac{dF_{u,n+1}(x_u)}{dx_u}+(1-\theta)(r-r_u)\frac{dS^{l}{}_{u,n}(x_u)}{dx_u}}{\left(1+(1-w)c+w(1-\theta)r\right)^{n+1}}+\frac{\frac{dV_{u,T}(x_u)}{dx_u}}{\left(1+(1-w)c+w(1-\theta)r\right)^{T}}\leq 0$$

Or:

$$\frac{dF_{u,0}(x_u)}{dx_u}+\sum_{n=0}^{T-1}\frac{\frac{dF_{u,n+1}(x_u)}{dx_u}+(1-\theta)(r-r_u)\frac{dS^{l}{}_{u,n}(x_u)}{dx_u}}{(1+k_w)^{n+1}}+\frac{\frac{dV_{u,T}(x_u)}{dx_u}}{(1+k_w)^{T}}\leq 0$$

The interpretation of this equation is straightforward: the net present value, determined according to the standard WACC method, of the last euro invested in the project u is negative or equal to zero. There is indeed optimum project dimensioning when the standard WACC method is used. A project for which $x_u = 0$ (i.e. an unrealized project), presents a negative or zero marginal net present value. The marginal net present value of a realized project is equal to zero.

The project selection model thus presented gives us the cash flow adjustment that allows us to evaluate the effect on the equity value of contracting an additional loan amount at a subsidized interest rate. In order to determine the economic value of the projects, and consequently the firm's value, we provide, in the appendix, a model of the problem examined under the form of a linear program in which the dimensions of the projects have already been decided. The operating cash flow and the maximum subsidized loan amounts thus appear on the right-hand side of the constraints. By using the property of strong duality in linear programming, we can move from marginal analysis, suitable for decision-making purposes, to an analysis of the firm's value. At the optimum financing allocation, the sum of the products of each right-hand-side coefficient by the associated dual variable is in fact equal to the value of the objective function. The firm's value is thus equal to the sum of the operating cash flows and the B-adjustments discounted at the weighted average cost of capital.

4. MODEL WITH A DEBT RATIO CONSIDERING THE ECONOMIC VALUE OF DEBT

The program is now written as follows:

$$Max \sum_{u=1}^{Z}\left(V_{u,0} + F_{u,0}\left(x_u\right) + B_{u,0} + S_{u,0}\right)$$

$$s.t. \begin{cases} (u=1,...,z)\ (n=0,...,T-1)\ \ S_{u,n} - S_{u,n}^{l}\left(x_u\right) \leq 0 \quad (14) \\ (u=1,...,z)\ (n=0,...,T-2)\ \ \left(1+c\right)V_{u,n} - V_{u,n+1} - B_{u,n+1} - S_{u,n+1} \\ \qquad\qquad + \left(1+\left(1-\theta\right)r\right)B_{u,n} + \left(1+\left(1-\theta\right)r_u\right)S_{u,n} - F_{u,n+1}\left(x_u\right) = 0 \quad (15) \\ (u=1,...,z)\ \ \left(1+c\right)V_{u,T-1} - V_{u,T}\left(x_u\right) - \left(1+\left(1-\theta\right)r\right)B_{u,T-1} - \left(1+\left(1-\theta\right)r_u\right)S_{u,T-1} - F_{u,T}\left(x_u\right) = 0 \quad (16) \\ (u=1,...,z)\ (n=0,...,T-1)\ \ \left(1+\left(1-\theta\right)r\right)Y_{u,n} - Y_{u,n+1} - \left(1+\left(1-\theta\right)r_u\right)S_{u,n} + S_{u,n+1} = 0 \\ (n=0,...,T-1)\ \ \left(1-w\right)\sum_{u=1}^{z}\left(B_{u,n} + Y_{u,n}\right) - w\sum_{u=1}^{z}V_{u,n} = 0 \quad (17) \\ (u=1,...,z)\ (n=0,...,T-1)\ \ B_{u,n} \geq 0 \quad (18) \\ (u=1,...,z)\ (n=0,...,T-1)\ \ S_{u,n} \geq 0 \quad (19) \\ (u=1,...,z)\ \ x_u \geq 0 \end{cases}$$

As previously, we will use Kuhn-Tucker's conditions to determine the values of the dual variables. The dual variables associated with the five types of constraints coopioed bitween equations (14) and (17) in the above pogrem will be respectively noted $\lambda_{u,n}$, $\pi_{u,n}$, $\pi_{u,T-1}$, $\gamma_{u,n}$ and μ_n.

The dual relations are then written:

$$\begin{cases} (u=1,...,z)\ (n=0,...,T-1)\ \ \lambda_{u,n} \geq 0 \\ (u=1,...,z)\ \ 1-\pi_{u,0}\left(1+c\right)+w\mu_0=0 \quad (20) \\ (u=1,...,z)\ \ 1-\lambda_{u,0}-\pi_{u,0}\left(1+\left(1-\theta\right)r_u\right)+\left(1+\left(1-\theta\right)r_u\right)\gamma_{u,0} \leq 0 \\ (u=1,...,z)\ \ 1-\pi_{u,0}\left(1+\left(1-\theta\right)r\right)-\left(1-w\right)\mu_0 \leq 0 \quad (21) \\ (u=1,...,z)\ \ -\left(1+\left(1-\theta\right)r\right)\gamma_{u,0}-\left(1-w\right)\mu_0 \leq 0 \\ (u=1,...,z)\ (n=1,...,T-1)\ \ -\pi_{u,n}\left(1+c\right)+\pi_{u,n-1}+w\mu_n=0 \\ (u=1,...,z)\ (n=1,...,T-1)\ \ -\lambda_{u,n}-\pi_{u,n}\left(1+\left(1-\theta\right)r_u\right)+\pi_{u,n-1}-\gamma_{u,n-1}+\left(1+\left(1-\theta\right)r_u\right)\gamma_{u,n} \leq 0 \\ (u=1,...,z)\ (n=1,...,T-1)\ \ -\left(1+\left(1-\theta\right)r\right)\gamma_{u,n}+\gamma_{u,n-1}-\left(1-w\right)\mu_n \leq 0 \\ (u=1,...,z)\ (n=1,...,T-1)\ \ -\pi_{u,n}\left(1+\left(1-\theta\right)r\right)+\pi_{u,n-1}-\left(1-w\right)\mu_n \leq 0 \\ (u=1,...,z)\ \ \dfrac{dF_{u,0}\left(x_u\right)}{dx_u}+\displaystyle\sum_{n=0}^{T-1}\left(\pi_{u,n}\frac{dF_{u,n+1}\left(x_u\right)}{dx_u}+\lambda_{u,n}\frac{dS^l_{u,n}\left(x_u\right)}{dx_u}\right)+\pi_{u,T-1}\frac{dV_{u,T}\left(x_u\right)}{dx_u} \leq 0 \quad (22) \end{cases}$$

As previously, we will assume that all type (14) constraints are binding. Loans are therefore contracted at rate *r*. There is thus at least one project *v* for which $B_{v,0} > 0$. For this project, the corresponding constraint (18) is not binding; the inequality (21) is therefore an equality which, associated with the equation (20), gives the following system:

$$1-\pi_{v,0}\left(1+\left(1-\theta\right)r\right)-\left(1-w\right)\mu_0=0$$
$$1-\pi_{v,0}\left(1+c\right)+w\mu_0=0$$

As previously, it follows that:

$$\pi_{v,0}=\frac{1}{1+\left(1-w\right)c+w\left(1-\theta\right)r}, \qquad \mu_0=\frac{c-\left(1-\theta\right)r}{1+\left(1-w\right)c+w\left(1-\theta\right)r}$$

According to equation (20), we have:

$$u = 1, \ldots, z \qquad \pi_{u,0} = \pi_{v,0} = \frac{1}{1 + (1-w)c + w(1-\theta)r}$$

Projects eligible for subsidized financing in year 0 have a non-binding type (19) constraint: $S_{u,0} > 0$. We therefore have:

$$1 - \lambda_{u,0} - \pi_{u,0}\left(1 + (1-\theta)r_u\right) + \left(1 + (1-\theta)r_u\right)\gamma_{u,0} = 0$$
$$-\left(1 + (1-\theta)r\right)\gamma_{u,0} - (1-w)\mu_0 = 0$$

$$\gamma_{u,0} = -\frac{(1-w)\left(c - (1-\theta)r\right)\pi_{u,0}}{1 + (1-\theta)r}$$

We obtain: $$\lambda_{u,0} = \frac{(1-\theta)(r - r_u)}{1 + (1-\theta)r}$$

For one additional euro borrowed at a subsidized rate available in year 0, the dual variable $\lambda_{u,0}$ can be interpreted as equal to the present value of the after-tax difference between the interest expense that would be paid at the non-subsidized rate *r* and that paid at the subsidized rate r_u. This difference is discounted at the after-tax cost of the non-subsidized loan. This is in fact the E-adjustment proposed by Myers (1974) in the case of adjusted present value.

We would remark once again that the variables $\pi_{u,0}$ represent the discount factor that must be used with the standard WACC method. By noting k_w the discount rate from year 0 to year 1, we have:

$$1 + k_w = \frac{1}{\pi_{u,0}} = 1 + (1-w)c + w(1-\theta)r$$

Again, k_w can be described as an after-tax weighted average cost of capital. An increase in the right-hand side of constraint (15) corresponds implicitly to an increase in the after-tax operating cash flow. As previously, the value of the objective function increases by an amount equal to the increase in operating cash flow discounted at the after-tax WACC.

We will now generalize these results to the following years. By a reasoning process similar to the one used in the preceding section, we obtain for any project, as before:

$$\pi_{u,n} = \frac{1}{\left(1+(1-w)c+w(1-\theta)r\right)^{n+1}} = \frac{1}{\left(1+k_w\right)^{n+1}}$$

Moreover, projects benefiting from a loan $S_{u,n}$ in year n have their constraint (14) binding. We then have:

$$-\lambda_{u,n} - \pi_{u,n}\left(1+(1-\theta)r_u\right) + \pi_{u,n-1} - \gamma_{u,n-1} + \left(1+(1-\theta)r_u\right)\gamma_{u,n} = 0$$

$$-\left(1+(1-\theta)r\right)\gamma_{u,n} + \gamma_{u,n-1} - (1-w)\mu_n = 0$$

In order to determine $\lambda_{u,n}$ as simply as possible, we will calculate by induction the value of $\gamma_{u,n} - \pi_{u,n}$.

$$\gamma_{u,n-1} - \left(1+(1-\theta)r\right)\gamma_{u,n} = (1-w)\mu_n = \pi_{u,n-1} - \pi_{u,n}\left(1+(1-\theta)r\right)$$

which gives:

$$\gamma_{u,n} - \pi_{u,n} = \frac{\gamma_{u,n-1} - \pi_{u,n-1}}{1+(1-\theta)r}$$

And then immediately:

$$\gamma_{u,n} - \pi_{u,n} = \frac{\gamma_{u,0} - \pi_{u,0}}{\left(1+(1-\theta)r\right)^n} = -\frac{1}{\left(1+(1-\theta)r\right)^{n+1}}$$

And finally:

$$\lambda_{u,n} = \left(1+(1-\theta)r_u\right)\left(\gamma_{u,n} - \pi_{u,n}\right) - \left(\gamma_{u,n-1} - \pi_{u,n-1}\right) = -\frac{1+(1-\theta)r_u}{\left(1+(1-\theta)r\right)^{n+1}} + \frac{1}{\left(1+(1-\theta)r\right)^n}$$

$$\lambda_{u,n} = \frac{(1-\theta)(r-r_u)}{\left(1+(1-\theta)r\right)^{n+1}}$$

The ability in year n to contract an additional euro at the subsidized rate r_u makes it possible to increase the sum of the net present values by an amount equal to the after-tax interest expense differential discounted at the after-tax cost of the non-subsidized loan. We thus encounter in the general case the E-adjustment proposed by Myers (1974) (reformulated by Myers and Pogue (1974)).

We can now replace the dual variables by their respective values in the equation (22):

$$\frac{dF_{u,0}\left(x_u\right)}{dx_u}+\sum_{n=1}^{T}\frac{\dfrac{dF_{u,n}\left(x_u\right)}{dx_u}}{\left(1+\left(1-w\right)c+w\left(1-\theta\right)r\right)^n}+\sum_{n=0}^{T-1}\frac{\left(1-\theta\right)\left(r-r_u\right)\dfrac{dS^l_{u,n}\left(x_u\right)}{dx_u}}{\left(1+\left(1-\theta\right)r\right)^{n+1}}+\frac{\dfrac{dV_{u,T}\left(x_u\right)}{dx_u}}{\left(1+\left(1-w\right)c+w\left(1-\theta\right)r\right)^T}\leq 0$$

Or:

$$\frac{dF_{u,0}\left(x_u\right)}{dx_u}+\sum_{n=1}^{T}\frac{\dfrac{dF_{u,n}\left(x_u\right)}{dx_u}}{\left(1+k_w\right)^n}+\sum_{n=0}^{T-1}\frac{\left(1-\theta\right)\left(r-r_u\right)\dfrac{dS^l_{u,n}\left(x_u\right)}{dx_u}}{\left(1+\left(1-\theta\right)r\right)^{n+1}}+\frac{\dfrac{dV_{u,T}\left(x_u\right)}{dx_u}}{\left(1+k_w\right)^T}\leq 0$$

5. MODEL WITH A DEBT RATIO CONSIDERING THE MARKET VALUE OF DEBT

The program is now written as follows:

$$Max\sum_{u=1}^{Z}\left(V_{u,0}+F_{u,0}\left(x_u\right)+B_{u,0}+S_{u,0}\right)$$

$$s.t.\begin{cases}(u=1,...,z)\ (n=0,...,T-1)\ \ S_{u,n}-S^l_{u,n}\left(x_u\right)\leq 0 \quad (23)\\ (u=1,...,z)\ (n=0,...,T-2)\ \ \left(1+c\right)V_{u,n}-V_{u,n+1}-B_{u,n+1}-S_{u,n+1}\\ \qquad +\left(1+\left(1-\theta\right)r\right)B_{u,n}+\left(1+\left(1-\theta\right)r_u\right)S_{u,n}-F_{u,n+1}\left(x_u\right)=0 \quad (24)\\ (u=1,...,z)\ \ \left(1+c\right)V_{u,T-1}-V_{u,T}\left(x_u\right)-\left(1+\left(1-\theta\right)r\right)B_{u,T-1}-\left(1+\left(1-\theta\right)r_u\right)S_{u,T-1}-F_{u,T}\left(x_u\right)=0 \quad (25)\\ (u=1,...,z)\ (n=0,...,T-1)\ \left(1+r\right)W_{u,n}-W_{u,n+1}-\left(1+r_u\right)S_{u,n}+S_{u,n+1}=0\\ (n=0,...,T-1)\ \ \left(1-w\right)\sum_{u=1}^{z}\left(B_{u,n}+W_{u,n}\right)-w\sum_{u=1}^{z}V_{u,n}=0 \quad (26)\\ (u=1,...,z)\ (n=0,...,T-1)\ \ B_{u,n}\geq 0\\ (u=1,...,z)\ (n=0,...,T-1)\ \ S_{u,n}\geq 0\\ (u=1,...,z)\ \ x_u\geq 0\end{cases}$$

As before, the dual variables associated with the five types of constraints apearing from equation (23) to equation (26) in the above pogrem will be respectively noted $\lambda_{u,n}$, $\pi_{u,n}$, $\pi_{u,T-1}$, $\gamma_{u,n}$ and μ_n. The dual relations are then written:

$$\begin{cases} (u=1,...,z)\ (n=0,...,T-1)\quad \lambda_{u,n} \geq 0 \\ (u=1,...,z)\quad 1-\pi_{u,0}\left(1+c\right)+w\mu_0=0 \\ (u=1,...,z)\quad 1-\lambda_{u,0}-\pi_{u,0}\left(1+\left(1-\theta\right)r_u\right)+\left(1+r_u\right)\gamma_{u,0}\leq 0 \\ (u=1,...,z)\quad 1-\pi_{u,0}\left(1+\left(1-\theta\right)r\right)-\left(1-w\right)\mu_0\leq 0 \\ (u=1,...,z)\quad -\left(1+r\right)\gamma_{u,0}-\left(1-w\right)\mu_0\leq 0 \\ (u=1,...,z)\ (n=1,...,T-1)\quad -\pi_{u,n}\left(1+c\right)+\pi_{u,n-1}+w\mu_n=0 \\ (u=1,...,z)\ (n=1,...,T-1)\quad -\lambda_{u,n}-\pi_{u,n}\left(1+\left(1-\theta\right)r_u\right)+\pi_{u,n-1}-\gamma_{u,n-1}+\left(1+r_u\right)\gamma_{u,n}\leq 0 \\ (u=1,...,z)\ (n=1,...,T-1)\quad -\left(1+r\right)\gamma_{u,n}+\gamma_{u,n-1}-\left(1-w\right)\mu_n\leq 0 \\ (u=1,...,z)\ (n=1,...,T-1)\quad -\pi_{u,n}\left(1+\left(1-\theta\right)r\right)+\pi_{u,n-1}-\left(1-w\right)\mu_n\leq 0 \\ (u=1,...,z)\quad \dfrac{dF_{u,0}\left(x_u\right)}{dx_u}+\displaystyle\sum_{n=0}^{T-1}\left(\pi_{u,n}\frac{dF_{u,n+1}\left(x_u\right)}{dx_u}+\lambda_{u,n}\frac{dS^l_{u,n}\left(x_u\right)}{dx_u}\right)+\pi_{u,T-1}\frac{dV_{u,T}\left(x_u\right)}{dx_u}\leq 0 \end{cases}$$

Following the same reasoning as above:

$$\mu_0=\frac{c-\left(1-\theta\right)r}{1+\left(1-w\right)c+w\left(1-\theta\right)r}$$

$$\pi_{u,0}=\frac{1}{1+\left(1-w\right)c+w\left(1-\theta\right)r}=\frac{1}{1+k_w}$$

With, for projects eligible for subsidized financing at year 0:

$$1-\lambda_{u,0}-\pi_{u,0}\left(1+\left(1-\theta\right)r_u\right)+\left(1+r_u\right)\gamma_{u,0}=0$$

$$-\left(1+r\right)\gamma_{u,0}-\left(1-w\right)\mu_0=0$$

We obtain:

$$\gamma_{u,0} = -\frac{(1-w)(c-(1-\theta)r)\pi_{u,0}}{1+r} \qquad \lambda_{u,0} = \frac{r-r_u}{1+r}\left(1-\frac{\theta}{1+k_w}\right)$$

As previously:

$$\pi_{u,n} = \frac{1}{\left(1+(1-w)c+w(1-\theta)r\right)^{n+1}} = \frac{1}{(1+k_w)^{n+1}}$$

Moreover, projects benefiting from a loan $S_{u,n}$ at year n have their constraint (23) binding, which allows us to write:

$$-\lambda_{u,n} - \pi_{u,n}\left(1+(1-\theta)r_u\right) + \pi_{u,n-1} - \gamma_{u,n-1} + (1+r_u)\gamma_{u,n} = 0$$

$$-(1+r)\gamma_{u,n} + \gamma_{u,n-1} - (1-w)\mu_n = 0$$

Giving us by induction:

$$\gamma_{u,n} = -(1-w)\sum_{k=1}^{n}\frac{\mu_k}{(1+r)^{n+1-k}} + \frac{\gamma_{u,0}}{(1+r)^n}$$

For $n \geq 2$:

$$\lambda_{u,n} = \frac{r-r_u}{(1+r)^{n+1}}\left(\left(1-\frac{\theta}{1+k_w}\right)\left(\left(\frac{1+r}{1+k_w}\right)^n - (r-r_u)\right) + \frac{k_w-(1-\theta)r}{1+k_w}\sum_{k=1}^{n-1}\left(\frac{1+r}{1+k_w}\right)^k\right)$$

The expression of $\lambda_{u,n}$ (and thus the M-adjustment) proves difficult to use in practice. It combines an interest expense differential discounted at the before-tax marginal cost of the loan and a relatively complex expression dependent on r, r_u, k_w and θ.

We would remark that if the model only included projects generating a constant operating cash flow over an infinite horizon with loans also repaid over an infinite horizon, then the value of the subsidized loan in the amount S_u would be: $\frac{r-r_u}{r}S_u$.

6. CONSISTENCY OF THE VARIOUS APPROACHES

Introductory remarks on the consistency of approaches

If one considers a firm that has just decided on its choice of projects and financing, then a valuation according to the four approaches - equity residual method, standard WACC method with B-adjustment, standard WACC method with E-adjustment and standard WACC method with M-adjustment - would necessarily arrive at the same result. The firm's value can in fact be described in four different ways, depending on whether one considers the shareholder's point of view or whether one assumes that its financing results from an optimization subject to the constraint of respecting a debt ratio expressed either in book value, economic value, or market value. We would comment that for a given financing strategy and in the presence of a subsidized loan, the debt ratios corresponding to these three valuation methods differ. To be convinced of the consistency of the four approaches, it is enough to notice that the three models presented have for objective function the net present value of equity, and therefore the same optimum value.

Numerical illustration

Let's take a simple example, that of a new firm comprised of a single project that should generate a constant annual after-tax operating cash flow over an infinite horizon and for which the expected value is equal to 20 million dollars. This firm contracts two loans at year 0:

- a business loan of 100 million dollars at the annual interest rate of 10%,
- a loan in the amount of 60 million dollars at the subsidized interest rate of 4% granted by a government agency providing aid for business creation.

It is stipulated that these loans will be repaid over an infinite period. The payments to creditors are therefore made up solely of interest expense, constant from one year to the next, and deductible from the firm's taxable income (subject to an income tax rate of 50%). In view of this data, the cost of equity, equal to the expected yield required by the firm's shareholders, is 15%. All data are in nominal terms. We will verify that the four approaches arrive at the same value for the firm.

Equity residual method

One simply sets out a sum of the flows to shareholders discounted at the cost of equity.

$$100+60+\frac{20-(0.5\times 0.1\times 100)-(0.5\times 0.04\times 60)}{0.15}=252 \quad \text{million dollars}$$

Standard WACC method considering the book value of loan (B-adjustment)

One must simultaneously deduce the firm's value V as well as the debt ratio w

calculated in consideration of the book value of the debt, giving us the following two equations:

$$\frac{20+0.5\times(0.1-0.04)\times 60}{(1-w)\times 0.15+w\times 0.5\times 0.1}=V$$

$$w=\frac{100+60}{V}$$

We obtain $w=0.635$ and $V=252$ million dollars.

Standard WACC method considering the economic value or market value of the loan

Since the loan is repaid over an infinite horizon, there is no difference between the economic value and market value of the debt.

The value V results from the sum of the value V' of the operating cash flows and of the value of the subsidized loan. The calculation of V' necessitates the calculation of the debt ratio w' calculated in consideration of the market value of the debt, giving us the following three equations:

$$\frac{20}{(1-w')\times 0.15+w'\times 0.5\times 0.1}=V'$$

$$w'=\frac{100+24}{V'}$$

$$V'+36=V$$

We obtain $w'=0.574$ and $V=252$

All these figures are summarized in table 1. The approaches used, even though they result in the same value for the firm, do not value the subsidized loan in the same way. The reference situation (i.e. the unavailability of a subsidized loan) is not implicitly the same. A valuation with the B-adjustment assumes that if the subsidized loan cannot be contracted, then a loan of $0.635\times\frac{20}{0.0865}$, or 146.82 million dollars would be contracted at the rate of 10%. The E-adjustment assumes that a loan of 124 million dollars would be contracted at the rate of 10%.

7. CONCLUSIONS

We show in this article that the value of a subsidized loan depends on the definition of the target debt ratio (or financial leverage) that must be respected on a company-wide basis (or on the set of projects in the same class of risk). We examine three different definitions of the subsidized-loan amounts to be considered to calculate this debt ratio: book value, economic value and market value. These three definitions result in the same value for a non-subsidized loan and lead to the same valuation of the operating cash flows, which are discounted at the firm's after-tax weighted average (marginal) cost of capital. To value a subsidized loan, each model leads to a specific adjustment. When the debt ratio is calculated with the economic value of the loans, the adjustment obtained is the same as the one proposed by Myers (1974) for the Adjusted Present Value. Once the loans allocated, the equity residual method, the standard WACC method with a B-adjustment, an E-adjustment and an M-adjustment naturally result in the same firm's value (as the firm's implicit debt ratio used to compute the weighted average cost of capital has a distinct value in each case). It has to be noted that a firm subject to the international accounting standards (IAS) would determine its total debt by considering subsidized loans at their market value (IAS 39 does not mention taxation as an input to determine the fair value), whereas a small firm may consider subsidized debt at its book value. In practice, this justifies the coexistence of several possible adjustments (and subsidized-loan valuation formulas) in the industry.

Table 1: Numerical illustration, overview of results

	Implicit debt ratio	Present value of operating cash flows	Value of the subsidized loan	Firm's value
Debt ratio defined with respect to book value of debt	0.635	231.2	20.8	252
Debt ratio defined with respect to market (or economic) value of debt	0.574	216	36	252

8. APPENDIX

Firm's value: a linear programming approach

We are no longer dealing with a project selection model: we consider that a number p of projects are going to be undertaken by the firm (in other words, the variables x_u are now fixed). For each project u and each year n, $S^l_{u,n}$, $F_{u,n+1}$ and $F_{u,T}+V_{u,T}$ are given. The loan amounts $B_{u,n}$ and $S_{u,n}$ to be allocated to the various projects each year represent the program's variables. We will show the calculation in the case of the first model only (maximization of the value of equity subject to the constraint of satisfying a debt ratio w considering the book value of the loan).

It is now written as follows:

$$Max \sum_{u=1}^{p}\left(V_{u,0}+B_{u,0}+S_{u,0}\right)$$

$$s.t. \begin{cases} (u=1,...,p)\ (n=0,...,T-1) \quad S_{u,n} \le S^l_{u,n} \\ (u=1,...,p)\ (n=0,...,T-1) \quad -B_{u,n} \le 0 \\ (u=1,...,p)\ (n=0,...,T-1) \quad -S_{u,n} \le 0 \\ (u=1,...,p)\ (n=0,...,T-2) \quad (1+c)V_{u,n}-V_{u,n+1}-B_{u,n+1}-S_{u,n+1} \\ \qquad\qquad +\left(1+(1-\theta)r\right)B_{u,n}+\left(1+(1-\theta)r_u\right)S_{u,n}=F_{u,n+1} \\ (u=1,...,p) \quad (1+c)V_{u,T-1}-\left(1+(1-\theta)r\right)B_{u,T-1}-\left(1+(1-\theta)r_u\right)S_{u,T-1}=F_{u,T}+V_{u,T} \\ (n=0,...,T-1) \quad (1-w)\sum_{u=1}^{p}\left(B_{u,n}+S_{u,n}\right)-w\sum_{u=1}^{p}V_{u,n}=0 \end{cases}$$

We will notate the dual variables of this program as in the body of the article. At the optimum financing allocation (assumed non-degenerate), by the strong duality of linear programs, we have:

$$\sum_{u=1}^{P}\left(V_{u,0}+B_{u,0}+S_{u,0}\right)=\sum_{u=1}^{p}\sum_{n=0}^{T-1}\left(\pi_{u,n}F_{u,n+1}+\lambda_{u,n}S^l_{u,n}\right)+\sum_{u=1}^{P}\pi_{u,T-1}V_{u,T}$$

If ΔV denotes the value creation for the shareholders, we have:

$$\Delta V = \sum_{u=1}^{P}\left(V_{u,0} + B_{u,0} + S_{u,0} - F_{u,0}\right)$$

$$\Delta V = -\sum_{u=1}^{P} F_{u,0} + \sum_{u=1}^{p}\sum_{n=0}^{T-1}\left(\pi_{u,n}F_{u,n+1} + \lambda_{u,n}S^{l}_{u,n}\right) + \sum_{u=1}^{P}\pi_{u,T-1}V_{u,T}$$

Or:

$$\Delta V = -\sum_{u=1}^{P} F_{u,0} + \sum_{u=1}^{p}\sum_{n=0}^{T-1}\left(\frac{F_{u,n+1} + (1-\theta)(r - r_u)S^{l}_{u,n}}{\left(1 + w(1-\theta)r + (1-w)c\right)^{n+1}}\right) + \sum_{u=1}^{P}\frac{V_{u,T}}{\left(1 + w(1-\theta)r + (1-w)c\right)^{T}}$$

The total value V of the firm is therefore:

$$V = \sum_{u=1}^{P} F_{u,0} + \Delta V = \sum_{u=1}^{p}\left(\sum_{n=0}^{T-1}\left(\frac{F_{u,n+1} + (1-\theta)(r - r_u)S^{l}_{u,n}}{\left(1 + w(1-\theta)r + (1-w)c\right)^{n+1}}\right) + \frac{V_{u,T}}{\left(1 + w(1-\theta)r + (1-w)c\right)^{T}}\right)$$

By similar reasoning, when the loan is specified in economic value:

$$\begin{aligned} V &= \sum_{u=1}^{P} F_{u,0} + \Delta V \\ &= \sum_{u=1}^{p}\left(\sum_{n=0}^{T-1}\left(\frac{F_{u,n+1}}{\left(1 + w(1-\theta)r + (1-w)c\right)^{n+1}} + \frac{(1-\theta)(r - r_u)S^{l}_{u,n}}{\left(1 + (1-\theta)r\right)^{n+1}}\right) + \frac{V_{u,T}}{\left(1 + w(1-\theta)r + (1-w)c\right)^{T}}\right) \end{aligned}$$

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