Weighting Two Quality Indexes In Valuation Theory: Survival Function And An Alternative Technique

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ABSTRACT

In this paper, we analyze the different weighting techniques used in the valuation theory to correct and fit the market value of an asset with respect to the appraisals from each quality index; particularly in the valuation method of the two functions, under both independence and dependence of two quality indexes of an asset, and we expand these techniques by their survival functions. Furthermore, we propose a new tool to weight two quality indexes based on the mode values of its marginal distribution functions, which extend the variety of the possible weighted models to approach the market value. Finally, we give an application of these weighted models in one example of land pricing, and thus the assessments of the land property according to each weighted model are proposed.

Keywords: valuation method, survival function, quality index, weighted model.

Ponderando Dos Índices De Calidad En Teoría De Valoración: Función De Supervivencia Y Una Técnica Alternativa

RESUMEN

En este trabajo, analizamos las diferentes técnicas de ponderación utilizadas en teoría de valoración para corregir y ajustar el valor de mercado de un bien con respecto a las valoraciones con cada uno de los índices de calidad; concretamente en el método de valoración de las dos funciones, bajo independencia y dependencia de dos índices de calidad de un bien, y ampliamos estas técnicas mediante sus funciones de supervivencia. Además, proponemos una nueva herramienta para ponderar dos índices de calidad basada en los valores modales de sus funciones de distribución marginales, extendiendo la variedad de los modelos ponderados posibles para aproximar el valor de mercado. Finalmente, damos una aplicación de estos modelos ponderados en un ejemplo de tasación de tierras, proponiéndose valoraciones de la finca según cada modelo ponderado.

Palabras clave: método de valoración, función de supervivencia, índice de calidad, modelo ponderado.
JEL Classification: C10; G12

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1. INTRODUCTION

In risk analysis, different procedures based on weighted probability models are usual tools to reduce loss of the assessments in multivariate scenarios. In particular, the weighted distribution functions have been widely used to correct and fit the market value of an asset with respect to the appraisals from each component of a multidimensional quality index, through the valuation method of the two functions.

In the literature, the beta distribution has been considered a suitable model in both, valuation method of the two functions and PERT methodology, because it provides a wide variety of probabilistic shapes on finite interval. However, the beta model has been labelled by Law and Kelton (1982) as a rough model in the absence of data for problems of assessment of risk and uncertainty; thus several authors have paid more attention to the study and generalization of distributions required in these situations, see, e.g. Williams (1992), Callejón, Pérez and Ramos (1996), Johnson (1997), Herrerías, García, Cruz and Herrerías (2001), Herrerías (2002) and (2006), van Dorp and Kotz (2002a) and (2002b), García and García (2003), Herrerías, García and Cruz (2003), García, Cruz and García (2005) and Vivo and Franco (2006).

Specifically, the *valuation method based on the two distribution functions* (VMTD) was introduced by Ballestero (1971) as valuation method of the two beta distributions, and extended by Caballer (1975) and Romero (1977); as well as the practical utility of this methodology when only small samples are available (e.g. Alonso and Lozano (1985)). It has been also used for different applications, such as the valuation of real estates, irrigation, trees, business, ...

This valuation method allows to appraise an asset under uncertainty, when the appraiser only makes available the pessimistic, optimistic and most likely values, which may be supplied by an expert judgement, such as in PERT methodology; for instance, see García, Cruz and Andújar (1999), García, Trinidad and Gómez (1999), Cruz, García and García (2002), García, Cruz and García (2002), García, Cruz and Rosado (2002), Herrerías (2002) and (2006), García and García (2003), García, Herrerías and García (2003) and García, Trinidad and García (2006). Moreover, the VMTD provides the same valuations using the survival functions instead of distribution functions when there is only one quality index; however, it supplies assessments less than the *valuation method based on the two survival functions* (VMTS) when there are more than one quality index, see e.g. Franco, Herrerías, Vivo and Callejón (2005) and Vivo (2005).

In anyway, the valuation method of the two functions unfortunately presents some disadvantages when it is considered greater information by more than one quality index of this asset, such as loss or profit with respect to the assessments from each quality index.

In order to reduce loss of the assessments in risk analysis, it is usual to consider probability models weighting the distinct components of the quality index. In particular, seeking to reduce the depreciation (appreciation) underwent in the market by the VMTD (VMTS) when two quality indexes are made available, weighted probability models based on the marginal distribution functions of the two-dimensional quality index are used, in both independence and dependence between its components, to correct and fit the market value of the asset.

In this setting, the weighted model of two quality indexes requires to determine the weights of each component, i.e., the coefficients α or p of one quality index, respectively, according to the independence or dependence between both indexes. Thus, Herrerías (2002) and (2006) and García and García (2003) analyze three ways to calculate these weights:

- Subjective (supplied by an expert judgment).
- Modal (relationship between the modal values).
- Econometric (fit by the linear models).

Furthermore, under independence of the two quality indexes, Herrerías (2002) and (2006) examines the use of the three procedures, with their advantages and disadvantages, respectively. However, under dependence of them, he comments that these techniques are reduced to two: subjective and econometric. Nevertheless, we have not found any motive to discard the modal technique under dependence, except the same ones as in the case of independence.

Remark that, in the subjective technique, the expert (appraiser) supplies the information about the weights of the indexes, and so, one might point out the inconvenient of subjectivity itself, see Herrerías (2002) and (2006).

Therefore, we analyze the modal and econometric techniques used in the valuation method of the two functions, under both independence and dependence of two quality indexes of an asset, and we go deeply into the weighting of the two indexes by a double way, which extends the variety of possible weighted models to approach the market value of the asset. Firstly, we expand the weighted models from the survival functions of the two quality indexes by the modal and econometric techniques, and secondly, we propose the modal-mean technique as an alternative tool to weight two quality indexes based on the mode values of its marginal functions.

Likewise, the different weighted models from two quality indexes are applied in the valuation method of the two functions (VMTD and VMTS) to approach the market value in one example of land pricing, and consequently, the assessments of the land property according to each weighted model are proposed.

For that, this paper is organized as follows. Section 2 briefly discusses the techniques to weight the distribution functions of two quality indexes. Section 3 expands these tools to the survival function of two quality indexes. Section 4 introduces a new technique to weight two quality indexes, which is free of the probability model of the market value. Section 5 shows an application in the valuation method of the two functions. Finally, the main comments and conclusions of this paper are summarized.

2. WEIGHTING TWO QUALITY INDEXES BY THEIR DISTRIBUTION FUNCTIONS

In this section, we describe the modal and econometric techniques used to weight two quality indexes in the valuation method of the two functions, which were introduced by their marginal distribution functions.

2.1. Econometric technique

To summarize the econometric technique in the weighting of the marginal distribution functions of two-dimensional quality index of an asset, we distinguish between independent and dependent components.

2.1.1.Two independent quality indexes

From two independent quality indexes, Herrerías (2002) and (2006) and García and García (2003) study the next weighted model given by the distribution function

$$F_{WD}(i_1,i_2) = F_1^{\alpha}(i_1)F_2^{1-\alpha}(i_2) \tag{1}$$

where F_i is the marginal distribution function of the i -th quality index, $i = 1,2$, and with joint distribution function $F_I(i_1,i_2)$.

In this context, the econometric technique is based on the estimation of the following regression model

$$\log F_V(v_t) = \alpha \log F_1(i_{1t}) + \beta \log F_2(i_{2t}) + u_t,\ t = 1,\ldots,n$$

where F_V is the distribution function of the market value V of the asset.

So, the estimation $\hat{\alpha}$ of the coefficient of the first quality index might be calculated by restricted least squares, i.e., for least squares involving the restriction $\alpha + \beta = 1$.

2.1.2.Two dependent quality indexes

On the other hand, under dependence, the weighted model

$$F_{WD}(i_1,i_2) = pF_1(i_1) + (1-p)F_2(i_2) \tag{2}$$

is analyzed by Herrerías (2002) and (2006) and García and García (2003), and using the econometric technique, the weight is determined by the next regression model

$$F_V(v_t) = pF_1(i_{1t}) + qF_2(i_{2t}) + u_t,\ t = 1,\ldots,n$$

taking into account the constraint $p+q=1$. Therefore, the parameter p of the first component is estimated by restricted least squares.

Remark that the main advantage of the valuation methods of the two functions (VMTD and VMTS) against other appraisal methods is the practical utility of this methodology when only small samples are available and the weakness of the other ones in the absence of data. It reduces the usefulness of the above regression models, and consequently, for the estimation of the weights by the previous econometric techniques, which attempt to improve the assessments by the best fit among the distributions. Besides, one can note the addition of errors, in the estimation of the weights and the fit of the probability models (market value and quality index).

2.2. Modal technique

In this subsection, we analyze the modal technique to find the weighted model from two quality indexes, which is based on the equality of the distribution functions in the modal values of the market value and two-dimensional quality index (see e.g. Herrerías (2002) and (2006) and García and García (2003)).

Firstly, the modal technique determines the weights by the following relationship between the distribution functions

$$F_V(m) = F_{\text{WD}}(m_1, m_2) \tag{3}$$

where F_{WD} is the distribution function of the weighted model from the marginal distribution functions of the two-dimensional quality index.

Let us see now the behaviour of the modal technique when both components of the quality index are independent, and secondly, when the components are dependent.

2.2.1.Two independent quality indexes

From two independent quality indexes, from (1) and (3) the weight of the first component is given by

$$F_V(m) = F_1^{\alpha}(m_1) F_2^{1-\alpha}(m_2)$$

or equivalently,

$$\frac{F_V(m)}{F_2(m_2)} = \left(\frac{F_1(m_1)}{F_2(m_2)}\right)^{\alpha} \tag{4}$$

where $0 < \alpha < 1$ represents the weight of the first component of the quality index.

If $F_1(m_1) = F_2(m_2)$, then (4) only makes sense when $F_V(m) = F_1(m_1) = F_2(m_2)$, which is a strong restriction over the asset modal market value. In this case, α might be any point within $(0,1)$.

If $F_1(m_1) \neq F_2(m_2)$, then the weight of the first component holds

$$\alpha = \frac{\log F_V(m) - \log F_2(m_2)}{\log F_1(m_1) - \log F_2(m_2)}$$

wherein we point out the following contradictory situations:

(i) When $F_V(m) < F_2(m_2) < F_1(m_1)$ then $\alpha < 0$

(ii) When $F_V(m) < F_1(m_1) < F_2(m_2)$ then $\alpha > 1$

Therefore, so that $\alpha \in (0,1)$, it is necessary to impose the following condition over the asset modal market value

$$F_V(m) \in [F_i(m_i)\ F_j(m_j)] \text{ with } i \neq j \in \{1,2\} \text{ such that } F_i(m_i) < F_j(m_j) \quad (5)$$

Besides, Ballestero and Rodríguez (1999) and Herrerías (2002) and (2006) point out that the modal valuation has not to correspond with the modal quality index. In spite of everything, we conclude with the following disadvantage for the modal technique by marginal distribution functions under independence: strong restrictions over the mode of the distribution function of the market value are required to get feasible weights.

2.2.1. Two dependent quality indexes

When the two-dimensional quality index has dependent components, from (2) and (3), the modal technique allows us to get the weights by

$$F_V(m) = pF_1(m_1) + (1-p)F_2(m_2)$$

or equivalently,

$$F_V(m) - F_2(m_2) = p\big(F_1(m_1) - F_2(m_2)\big) \quad (6)$$

where $0 < p < 1$ represents the weight of the first component of the quality index.

If $F_1(m_1) = F_2(m_2)$, then (6) only makes sense for $F_V(m) = F_1(m_1) = F_2(m_2)$, which is a strong restriction over the asset modal market value, and thus, p could be chosen within $(0,1)$.

If $F_1(m_1) \neq F_2(m_2)$, then the weight holds

$$p = \frac{F_V(m) - F_2(m_2)}{F_1(m_1) - F_2(m_2)}$$

wherein we point out the following contradictory cases:

(iii) When $F_V(m) < F_2(m_2) < F_1(m_1)$ then $p < 0$

(iv) When $F_V(m) < F_1(m_1) < F_2(m_2)$ then $p > 1$

Therefore, so that $p \in (0,1)$, it is necessary to impose the condition (5) over the modal market value.

Consequently, under dependence between both quality indexes, the disadvantages for the modal technique by marginal distribution functions are the same as in the independent case. Thus, there is not other motive to discard the modal technique when both quality indexes are dependent.

3. WEIGHTING TWO QUALITY INDEXES BY THEIR SURVIVAL FUNCTIONS

In this section, we discuss the modal and econometric techniques from an alternative viewpoint, using the marginal survival functions of the two-dimensional quality index, which expand the different weighting used in the valuation method of the two functions.

3.1. Econometric technique

Let us see now the econometric technique to weight two quality indexes through their marginal survival functions, instead of their distribution functions. For that, we also consider the independent and dependent cases.

3.1.1.Two independent quality indexes

Assuming independence between the components of the quality index, the weighted model from marginal survival functions is given by the survival function

$$S_{WS}(i_1, i_2) = S_1^{\alpha}(i_1) S_2^{1-\alpha}(i_2) \tag{7}$$

where $S_i = 1 - F_i$ is the marginal survival function of the i -th quality index, $i = 1,2$, and with joint survival function $S_I(i_1, i_2)$.

Thus, the estimation $\hat{\alpha}$ can be obtained through restricted least squares in the following regression model

$$\log S_V(v_t) = \alpha \log S_1(i_{1t}) + \beta \log S_2(i_{2t}) + u_t,\ t = 1,\ldots,n$$

taking into account the constraint $\alpha + \beta = 1$, where $S_V = 1 - F_V$ is the survival function of the market value of the asset.

3.1.2.Two dependent quality indexes

Likewise, when the components of the quality index are dependent, the weighted model from marginal survival functions is given by

$$S_{WS}(i_1, i_2) = pS_1(i_1) + (1 - p)S_2(i_2) \tag{8}$$

and hence, the estimation $\hat{p}$ might be calculated by restricted least squares in the next regression model

$$S_V(v_t) = pS_1(i_{1t}) + qS_2(i_{2t}) + u_t,\ t = 1,\ldots,n$$

using the constraint $p + q = 1$.

Remark that the econometric technique based on the marginal survival functions has the same criticism as one based on the marginal distribution functions to estimate the weights of two quality indexes in the valuation methods of the two functions (VMTD and VMTS).

3.2. Modal technique

Let us see now an alternative viewpoint in the modal technique based on the survival functions to weight two quality indexes, instead of the distribution functions.

Thus, the weights with the modal technique are determined by the equality of the survival functions in the modes of the market value and two-dimensional quality index

$$S_V(m) = S_{WS}(m_1, m_2) \tag{9}$$

where S_{WS} is the survival function of the weighted model from the marginal survival functions of the two-dimensional quality index.

To discuss the behaviour of this weighting technique, the independent and dependent cases are also studied.

3.2.1.Two independent quality indexes

In this case, from (7) and (9), the weight of the first component is given by

$$S_V(m) = S_1^{\alpha}(m_1) S_2^{1-\alpha}(m_2)$$

or equivalently,

$$\frac{S_V(m)}{S_2(m_2)} = \left(\frac{S_1(m_1)}{S_2(m_2)}\right)^{\alpha} \qquad (10)$$

If $S_1(m_1) = S_2(m_2)$, then (10) only makes sense for $S_V(m) = S_1(m_1) = S_2(m_2)$, which is a strong restriction over the asset modal market value, and α might be any point within $(0,1)$.

If $S_1(m_1) \neq S_2(m_2)$, then the weight holds

$$\alpha = \frac{\log S_V(m) - \log S_2(m_2)}{\log S_1(m_1) - \log S_2(m_2)}$$

wherein we remark the following contradictory situations:

(v) When $S_V(m) > S_2(m_2) > S_1(m_1)$ then $\alpha < 0$

(vi) When $S_V(m) > S_1(m_1) > S_2(m_2)$ then $\alpha > 1$

Therefore, so that $\alpha \in (0,1)$, it is necessary to impose the following condition over the asset modal market value

$$S_V(m) \in [S_i(m_i)\ S_j(m_j)] \text{ with } i \neq j \in \{1,2\} \text{ such that } S_i(m_i) < S_j(m_j) \qquad (11)$$

Remark that the disadvantage of the modal technique from marginal survival functions is the strong restriction (11) over the mode of the survival function of the market value in order to obtain feasible weights.

3.2.1.Two dependent quality indexes

When both quality indexes are dependent, from (8) and (9) the modal technique allows us to obtain the weights by

$$S_V(m) = pS_1(m_1) + (1-p)S_2(m_2)$$

or equivalently,

$$S_V(m) - S_2(m_2) = p(S_1(m_1) - S_2(m_2)) \qquad (12)$$

where $0 < p < 1$ is the weight of the first quality index.

If $S_1(m_1) = S_2(m_2)$, then (12) only makes sense for $S_V(m) = S_1(m_1) = S_2(m_2)$, which is a strong restriction over the asset modal market value, but p can be any point within $(0,1)$.

If $S_1(m_1) \neq S_2(m_2)$, then the weights holds

$$p = \frac{S_V(m) - S_2(m_2)}{S_1(m_1) - S_2(m_2)}$$

wherein we point out the following contradictory cases:

(vii) When $S_V(m) > S_2(m_2) > S_1(m_1)$ then $p < 0$

(viii) When $S_V(m) > S_1(m_1) > S_2(m_2)$ then $p > 1$

Therefore, so that $p \in (0,1)$, it is necessary to impose the condition (11) over the modal market value.

So, the disadvantages for the modal technique from the marginal survival functions of two dependent quality indexes are the same as in the independent case.

Besides, note that $S_1(m_1) = S_2(m_2)$ if and only if $F_1(m_1) = F_2(m_2)$, and the conditions (5) and (11) are equivalent. Consequently, we have found the same criticisms for this technique through both ways, the marginal survival functions and the marginal distribution functions.

4. MODAL-MEAN TECHNIQUE TO WEIGHT TWO QUALITY INDEXES

In this section, we discuss a new technique to find the weights of the components, avoiding the disadvantages of the former tools: the subjectivity, the weakness of the econometric methods and the restrictions over the modal market value of the modal technique.

For that, we consider a method based on the marginals of the bivariate probability model of the two-dimensional quality index and the weighted model from them, to weight both components. So, the modal-mean technique provides weighted models from both quality indexes and free of the market value distribution, because it is only based on the modal values of the quality indexes.

4.1. Modal-mean technique by distribution functions

In order to reduce the depreciation of the VMTD with respect to the assessments from each component, we analyze the weighting of the marginal distribution functions of two-dimensional quality index.

Remark that for any weight of the first component, $0 < \alpha < 1$ or $0 < p < 1$ under independence or dependence, respectively, the weighted models (1) and (2) are bound by the marginal distribution functions of $F_I(i_1, i_2)$

$$\inf\{F_1(i_1), F_2(i_2)\} \leq F_{WD}(i_1, i_2) \leq \sup\{F_1(i_1), F_2(i_2)\}$$

for all quality index (i_1, i_2). In particular, for the modal quality index (m_1, m_2), the following inequalities hold

$$\inf\{F_1(m_1), F_2(m_2)\} \leq F_{WD}(m_1, m_2) \leq \sup\{F_1(m_1), F_2(m_2)\}$$

In this setting, we propose the modal-mean technique to obtain the weight (α or p) such that the distance among these three values is minimized, i.e., the modal value of the weighted distribution function and the two modal values of the marginal distribution functions, which is given by

$$F_{WD}(m_1, m_2) = \frac{F_1(m_1) + F_2(m_2)}{2} \qquad (13)$$

Note that the modal-mean technique uses the available information by these quality indexes; so it is not influenced by the market value, and consequently, this procedure does not require any restriction over the modal market value.

4.1.1.Two independent quality indexes

Firstly, when the components of the quality index are independent, from (1) and (13), the weight α by the modal-mean method is given by

$$F_1^{\alpha}(m_1)F_2^{1-\alpha}(m_2) = \frac{F_1(m_1) + F_2(m_2)}{2}$$

or equivalently,

$$\left(\frac{F_1(m_1)}{F_2(m_2)}\right)^{\alpha} = \frac{F_1(m_1) + F_2(m_2)}{2F_2(m_2)}$$

where $0 < \alpha < 1$ represents the weight of the first component of the quality index.

If $F_1(m_1) = F_2(m_2)$, then α can take any value within $(0,1)$.

If $F_1(m_1) \neq F_2(m_2)$, then the weight holds

$$\alpha = \frac{\log\left(\frac{F_1(m_1) + F_2(m_2)}{2}\right) - \log F_2(m_2)}{\log F_1(m_1) - \log F_2(m_2)} \in (0,1)\,.$$

Therefore, this method always provides feasible weights from two independent quality indexes.

4.1.2.Two dependent quality indexes

On the other hand, when the components of the quality index are dependent, from (2) and (13), we have the next equation

$$pF_1(m_1) + (1-p)F_2(m_2) = \frac{F_1(m_1) + F_2(m_2)}{2}$$

or equivalently,

$$p\big(F_1(m_1) - F_2(m_2)\big) = \frac{F_1(m_1) - F_2(m_2)}{2}$$

where $0 < p < 1$ represents the weight of the first quality index.

If $F_1(m_1) = F_2(m_2)$, then p may be any point within $(0,1)$.

If $F_1(m_1) \neq F_2(m_2)$, then $p = 1/2$.

Remark that, under dependence between the components, the modal-mean technique provides the same weight for both quality indexes, which shows its coherence, since the dependent structure between the components includes the predominance and importance of one over the other, and therefore it will be contradictory the assignment of the different coefficients in the weighted model.

4.2. Modal-mean technique by survival functions

Let us see now the weighting of the marginal survival functions of two-dimensional quality index.

Taking into account that for any weight of the first component, $0 < \alpha < 1$ or $0 < p < 1$ under independence or dependence, respectively, the weighted models (7) and (8) are bound by the marginal survival functions of $S_I(i_1, i_2)$

$$\inf\{S_1(i_1)\ S_2(i_2)\} \leq S_{WS}(i_1, i_2) \leq \sup\{S_1(i_1)\ S_2(i_2)\}$$

for all quality index (i_1, i_2). Hence, the weighted survival function S_{WS} allows to reduce the appreciation of the VMTS with respect to the assessments from each component.

In particular, for the modal quality index (m_1, m_2), we have

$$\inf\{S_1(m_1)\ S_2(m_2)\} \leq S_{WS}(m_1, m_2) \leq \sup\{S_1(m_1)\ S_2(m_2)\}$$

and therefore, the coefficient (α or p) by the modal-mean technique is defined by the minimum distance among the modal value of the weighted model and the two modal values of the marginal survival functions

$$S_{WS}(m_1, m_2) = \frac{S_1(m_1) + S_2(m_2)}{2} \tag{14}$$

Remark that (14) only uses the two modal values of the marginal survival functions, just like the modal-mean method from the marginal distribution functions, and consequently, it does not require any restriction over the modal market value.

4.2.1.Two independent quality indexes

In the first place, when the components of the quality index are independent, from (7) and (14), the weight α of the first component is determined by

$$S_1^{\alpha}(m_1) S_2^{1-\alpha}(m_2) = \frac{S_1(m_1) + S_2(m_2)}{2}$$

or equivalently,

$$\left(\frac{S_1(m_1)}{S_2(m_2)}\right)^{\alpha} = \frac{S_1(m_1)+S_2(m_2)}{2S_2(m_2)}$$

If $F_1(m_1)=F_2(m_2)$, i.e. $S_1(m_1)=S_2(m_2)$, then α can be any value within $(0,1)$.
If $F_1(m_1)\neq F_2(m_2)$, i.e. $S_1(m_1)\neq S_2(m_2)$, then the weight holds

$$\alpha = \frac{\log\left(1-\dfrac{F_1(m_1)+F_2(m_2)}{2}\right)-\log(1-F_2(m_2))}{\log(1-F_1(m_1))-\log(1-F_2(m_2))} \in (0,1)$$

Therefore, this method always provides feasible weights from two independent quality indexes.

4.2.2.Two dependent quality indexes

In the second place, when both quality indexes are dependent, from (8) and (14), the weight p of the first component is given by

$$pS_1(m_1)+(1-p)S_2(m_2) = \frac{S_1(m_1)+S_2(m_2)}{2}$$

or equivalently,

$$p(S_1(m_1)-S_2(m_2)) = \frac{S_1(m_1)-S_2(m_2)}{2}$$

If $F_1(m_1)=F_2(m_2)$, then p can be chosen within $(0,1)$.
If $F_1(m_1)\neq F_2(m_2)$, then $p=1/2$.

Consequently, under dependence between the components, the modal-mean technique provides the same weight for both marginal survival functions, just like the modal-mean method from the marginal distribution functions, and therefore it will be contradictory the assignment of the different coefficients in the weighted model, because its dependence includes the prevalence of one over the other.

5. APPLICATION IN THE VALUATION METHOD OF THE TWO FUNCTIONS

In this section, we apply these weighting techniques in one example of land pricing through the valuation method of the two functions when the appraiser only makes available the pessimistic, optimistic and most likely values of both, market value and two-dimensional quality index.

In particular, we consider the transactions of agricultural properties in *Tierras de Campos* and *Centro* regions (Valladolid, Spain) given in Alonso and Lozano (1985) and García and García (2003). The quality indexes used to explain the market values (€) are the income per hectare (*€/Ha*) and the inverse distance to Valladolid (1/*km*).

Table 1 displays data of the pessimistic, optimistic and most likely values for each variable (market value and two quality indexes). Here, the objective is to appraise an agricultural property whose incomes per hectare are 194.31€ and its location is 24*km* to Valladolid.

Table 1. Transactions of agricultural plots.

	Pessimist	Optimist	Most likely
V=Market value (*€/Ha*)	1502.53	3005.06	1953.29
I_1=Income (*€/Ha*)	120.20	300.51	195.33
I_2=Inverse distance (1/*km*)	1/70	1/10	1/50

Remark that García and García (2003) apply the VMTD in this example by the assumption of independence between both quality indexes and triangular distributions, as well as a triangular market value distribution. Moreover, they use the weighting techniques based on the distribution functions (subjective, modal and econometric) to reduce the depreciation of the VMTD with respect to the appraisals obtained from each component. In particular, $\alpha = 0.75$ is assumed as the subjective weight of the first component (incomes per hectare).

In this context, we will consider triangular and trapezoidal models for the market value of agricultural plots, i.e.

$$V \sim Triang(a_V = 1502.53, b_V = 3005.06, m_V = 1953.29)$$

and

$$V \sim Trap(a_V = 1502.53, b_V = 3005.06, m_V = 1953.29, w_V = 200.34)$$

where the trapezoidal model is obtained by the procedure of Vivo and Franco (2006) to determine the underlying distribution from the three values of Table 1, and thus, m_V is the midpoint of the modal interval $(m_1 = m_V - w_V, m_2 = m_V + w_V)$ and w_V is the half of its length.

These market value distributions are a sample of the different models that might be considered to obtain the assessments v_{D*} and v_{S*} of the land through both valuation methods, VMTD and VMTS, respectively, which are defined by

$$v_{D*} = \phi_D(i_1, i_2) = F_V^{-1} \circ F_*(i_1, i_2) \tag{15}$$

and

$$v_{S*} = \phi_S(i_1, i_2) = S_V^{-1} \circ S_*(i_1, i_2) \tag{16}$$

where asterisk is replaced by “WD” or “WS” according to the weighted model of both quality indexes, see e.g. Herrerías (2002) and (2006), García and García (2003) and Vivo (2005).

Note that the modal-mean technique is not influenced by the market value distribution, and so, the same coefficient has been assumed when the market value has a trapezoidal distribution for a better comparison between the assessments of both distributions.

In the first place, the weights of the first component of the two-dimensional quality index have been determined by the different techniques based on their marginal distribution functions. Table 2 displays these weights and the appraisals v_{DWD} and v_{SWD} of the land through both methods, VMTD and VMTS, respectively, when each weighted model given by (1) is used in (15) and (16).

Table 2. Valuations by the weighted model of the marginal distribution functions (F_{WD}).

Weighting technique	α	F_V	v_{DWD}	v_{SWD}
Subjective	0.75	Triangular	2054.33	2655.11
Subjective	0.75	Trapezoidal	2035.06	2650.70
Econometric	0.615456	Triangular	2064.94	2609.91
Econometric	0.615456	Trapezoidal	2047.14	2604.92
Modal	0.82074	Triangular	2048.92	2695.77
Modal	0.82074	Trapezoidal	2028.85	2691.87
Modal-mean	0.702754	Triangular	2058.01	2635.08
Modal-mean	0.702754	Trapezoidal	2039.26	2630.41

Analogously, Table 3 shows the weights of the first component from (7) by the different techniques based on their marginal survival functions, and the assessments v_{DWS} and v_{SWS} of the property through both methods, VMTD and VMTS, respectively, when each weighted survival function S_{WS} is used in (15) and (16).

Table 3. Valuations by the weighted model of the marginal survival functions (S_{WS}).

Weighting technique	α	F_V	v_{DWS}	v_{SWS}
Subjective	0.75	Triangular	1689.99	2057.37
Subjective	0.75	Trapezoidal	1659.78	2038.54
Econometric	0.671763	Triangular	1705.06	2064.05
Econometric	0.671763	Trapezoidal	1672.43	2046.13
Modal	0.612085	Triangular	1712.13	2069.12
Modal	0.612085	Trapezoidal	1678.36	2051.86
Modal-mean	0.441782	Triangular	1714.64	2083.42
Modal-mean	0.441782	Trapezoidal	1680.46	2067.85

Remark that, in all cases, the VMTS proposes appraisals greater than the VMTD.

On the other hand, to give a more clear exposition of the behaviour of the modal-mean technique in both valuation methods, VMTD and VMTS, we show some graphs corresponding to the trapezoidal distribution for the market value and two independent and triangular quality indexes.

Note that to make easy the interpretation of these graphs in the plane, both quality indexes have been simultaneously taken from the least to the supreme into their supports, and the valuations from each weighting technique, modal, econometric and modal-mean, will be marked by “mo”, “ec” and “mm”, respectively.

In this setting, we represent the assessments from the weighted model based on the marginal distribution functions by the modal-mean technique, i.e. F_{WD} given by (1) with $\hat{\alpha} = 0.702754$. Figure 1 depicts these appraisals through both valuation methods, v_{DWD_m} and v_{SWD_m} , jointly with the ones obtained from the original distribution model F_I, as well as the others from each component of the quality index, v_1 and v_2 .

Thus, Figure 1 shows the depreciation (appreciation) underwent in the market value by the VMTD (VMTS) when two quality indexes are made available, with respect to the assessments from each quality index. However, the modal-mean technique through the weighted model of their marginal distribution functions provides greater assessments in each valuation method. Further, they are always between the appraisals from each component by the VMTD, and they are the greatest by the VMTS.

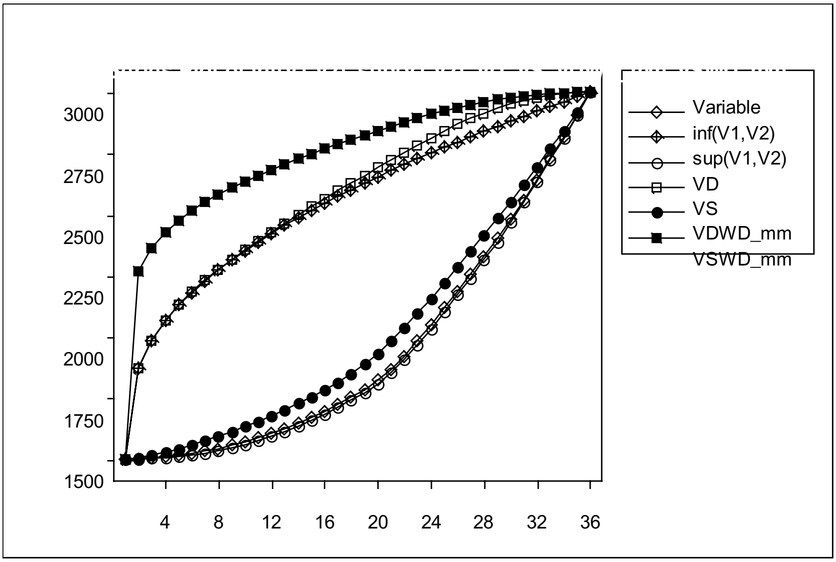

Figure 1. Valuations from the modal-mean technique by distribution functions.

Likewise, Figure 2 shows the valuations from the weighted models based on the marginal distribution function by the three weighting techniques, modal, econometric and modal-mean, i.e. F_{WD} given by (1) with $\hat{\alpha} = 0.82074$, 0.615456 and 0.702754 , respectively. This graph compares these appraisals from the modal-mean technique by both valuation methods, v_{DWD_m} and v_{SWD_m} , with respect to the ones from the others, since they are always between the assessments from the modal and econometric techniques in each valuation method, VMTD and VMTS.

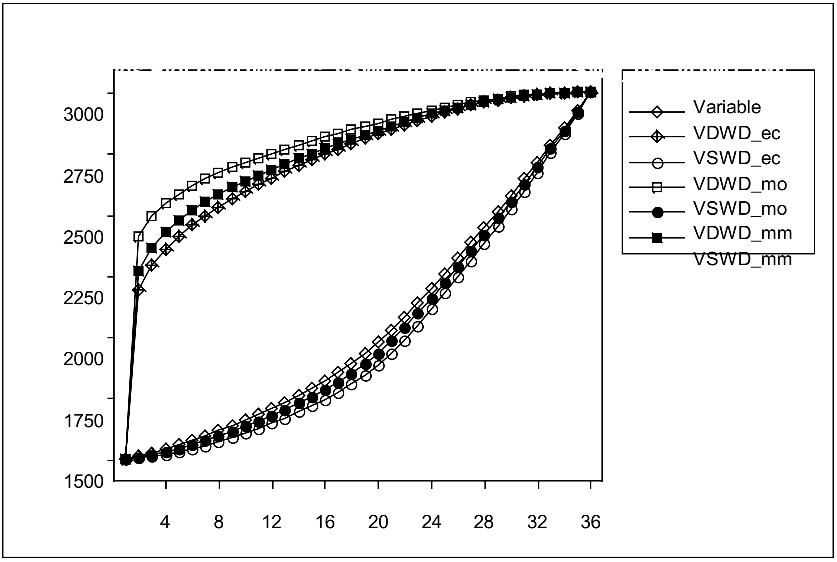

Figure 2. Valuations from the three weighting techniques by distribution functions.

In a similar way, we draw the valuations from the weighted model based on the marginal survival functions by the modal-mean technique, i.e. S_{WS} defined by (7) with $\hat{\alpha} = 0.441782$. Moreover, these appraisals through both valuation methods, v_{DWS_m} and v_{SWS_m}, are drawn jointly with the ones obtained from the original survival function S_I, as well as the others from each component of the quality index, v_1 and v_2.

So, Figure 3 describes the appreciation (depreciation) underwent in the market value by the VMTS (VMTD) when two quality indexes are made available, with respect to the assessments from each quality index. Likewise, the modal-mean technique through the weighted model of their marginal survival functions provides lower assessments in each valuation method. Further, they are always between the appraisals from each component by the VMTS, and they are the least by the VMTD.

Finally, Figure 4 depicts the valuations from the weighted models based on the marginal survival function by the three weighting techniques, modal, econometric and modal-mean, i.e. S_{WS} given by (7) with $\hat{\alpha} = 0.612085$, 0.671763 and 0.441782, respectively. It compares these appraisals from the modal-mean technique by both valuation methods, v_{DWS_m} and v_{SWS_m}, with respect to the ones from the others, since they are always over (below) the assessments from the modal and econometric techniques in the VMTS (VMTD).

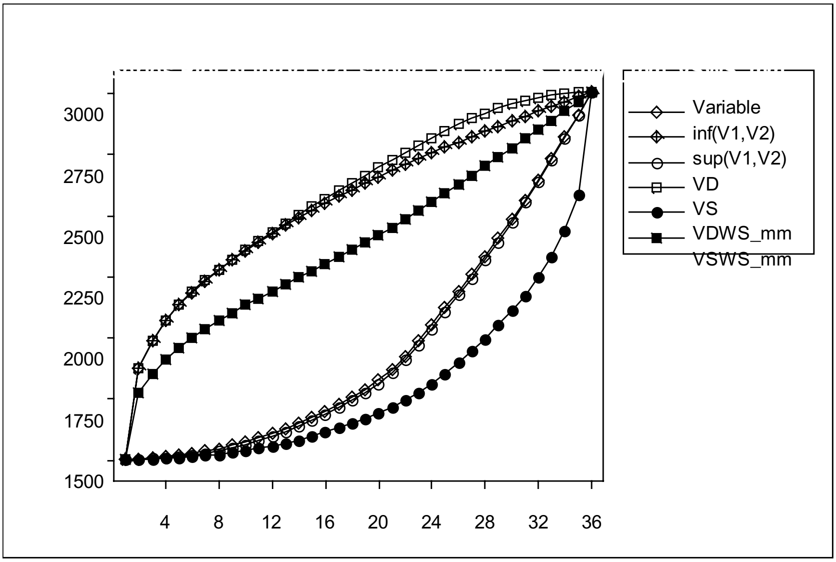

Figure 3. Valuations from the modal-mean technique by survival functions.

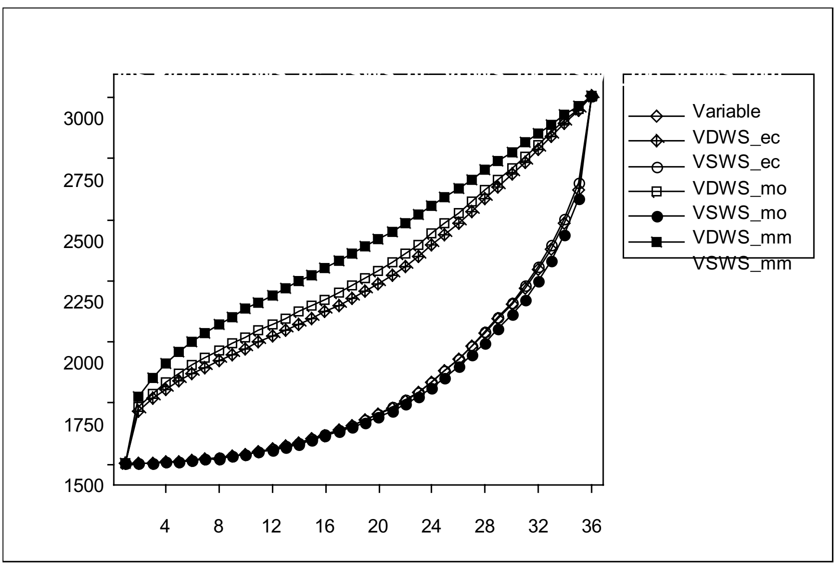

Figure 4. Valuations from the three weighting techniques by survival functions.

6. COMMENTS AND CONCLUSIONS

In this paper, the problem of the depreciation (appreciation) underwent by the valuation method of the two distribution (survival) functions has been considered when the appraiser only makes available the pessimistic, optimistic and most likely values of both variables, market value of an asset and its two-dimensional quality index.

In this case, the valuation of the asset is usually approached by weighted models of the two quality indexes, using their marginal distribution functions, and the used weighting techniques are subjective, modal and econometric, although the subjective technique is not recommended by itself subjectivity of the expert judgment.

Firstly, the marginal survival functions of two-dimensional quality index have been used to expand the variety of weighted models to approach the valuation of the asset.

So, strong restrictions over the mode of the survival function of the market value are required to get feasible weights by the modal technique, in both cases, independence and dependence between the components of the quality index. These restrictions are determined by the marginal survival functions in the modal quality index.

Also, these constraints are demanded to obtain feasible weights with the modal technique by the marginal distribution functions, which are equivalent to use the distribution functions instead of survival functions.

Likewise, the econometric technique estimates the weights of two quality indexes by regression models based on their marginal survival functions, which attempt to improve the assessments by the best fit among the survivals. Thus, its usefulness is reduced when only small samples are available; because this is the main advantage of the valuation methods of the two functions against other appraisal methods. Besides, one can note the addition of errors, in the estimation of the weights and the fit of the probability models (market value and quality index).

In order to avoid the disadvantages of these methods, the modal-mean technique is based on the minimum distance among the modal value of the weighted model and the two modal values of the marginal distribution (survival) functions.

Thus, this method is only based on the available information by these quality indexes, i.e., it is not influenced by the market value, and consequently, this procedure does not require any restriction over the modal market value. So, it allows find the weights of the components without the subjectivity, the weakness of the econometric methods and the restrictions over the mode of the modal technique.

Besides, the modal-mean technique always provides feasible weights from two independent quality indexes, which has been applied to appraise an agricultural land, and the behaviour of its valuations has been compared with ones of the other techniques.

Finally, under dependence between the components, the modal-mean technique provides the same weight for both quality indexes, which shows its coherence, since the dependent structure between the components includes the predominance and importance of one over the other, and therefore it will be contradictory the assignment of the different coefficients in the weighted model.

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