Empirical Insights on the Heterogeneity of the Spanish Stock Market

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ABSTRACT

We introduce the concept of cross-sectional heterogeneity (and homogeneity) of equity markets. From period to period, the stocks in a market may move more or less 'in line'. When stock prices move perfectly in line, we use the term 'homogeneous market'. When the stock price developments are not perfectly in line, we use the term 'heterogeneous markets' and we propose a measure for the degree of heterogeneity. This heterogeneity measure can be applies to all usual stock characteristics, such as stock returns and risks. We present empirical results on the development of the heterogeneity of the large cap segment of the Spanish stock market over time, thus providing some new insights.

Keywords: market description, benchmarking, performance evaluation.

Un Análisis Empírico De La Heterogeneidad Del Mercado De Capitales Español

RESUMEN

En este trabajo se introducen los conceptos de heterogeneidad y homogeneidad de los mercados de capitales. Partiendo de que periodo a periodo, las acciones se mueven en el mercado más o menos en línea, diremos que el mercado es homogéneo cuando los precios evolucionan perfectamente en línea y que es más o menos heterogéneo en caso contrario. Igualmente se propone una medida para determinar el grado de heterogeneidad que se podrá aplicar tanto al análisis del rendimiento como del riesgo. Por último se presenta una aplicación práctica para el mercado continuo español, concretamente para los activos que componen su índice de referencia.

Palabras clave: descripción del mercado, rendimiento, benchmarking.

1. INTRODUCTION

In this article we employ a new way of looking at financial markets as applied to the IBEX 35® index.[1] The distinctive feature of this approach is that we choose the perspective of available portfolio opportunities. When aiming at specific investment objectives and satisfying specific investment constraints, a universe of feasible portfolios can be identified. We label such a universe as a *portfolio opportunity set*. This set offers a level playing field for any portfolio that aims for the same investment objectives and respects the same constraints. Next, we observe the dynamics of each feasible portfolio in this opportunity set. Finally, we condense the information of the whole portfolio opportunity set into a compact and informative market description. The advantage of our approach is that such a market description can be made commensurate with any specific investment environment (such as an investment mandate as defined by investment goals and constraints).

We zoom in on a particular element of this approach, the *heterogeneity* of the portfolio opportunity set, which is a measure of the cross-sectional diversity across the available portfolio alternatives. This diversity can relate to the returns of the portfolios implied by the opportunity set, to the risk or any other performance measure defined for these portfolios. This heterogeneity of the portfolio opportunity set is a new concept, which produces additional insights into the nature of market, which is the domain for the portfolio concerned. Especially interesting the concept of market heterogeneity/homogeneity is for performance evaluation. In particular, the heterogeneity helps to quantify the effect of portfolio flexibility on the performance. Furthermore, as the professional quality of an active manager is judged on the basis of excess returns with respect to a (passive) benchmark or a reference index, we would like to know how relative are these excess returns (or risk-adjusted metrics such as information ratio). That is, we would like to know how difficult it was to achieve the attained over-performance over a given period. The heterogeneity measure can be used to "normalize" the performance(s) by dividing the relative performance values by the corresponding heterogeneity measure. Thus, we obtain performance values, which are "cleaned" from the contemporary market effects. Other applications can be derived easily. In this article, we especially concentrate on the development of the heterogeneity of the Spanish stock market, both in terms of returns and in terms of risk.

The organization of the paper is as follows. Section 2 reviews the portfolio opportunity set methodology and outlines additional methods of analysis, which are relevant for this study. Also it describes our data set. Section 3 provides empirical

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1   IBEX 35® is a registered trademark of Sociedad de Bolsas S.A. It is written with the symbol ® at the first occurrence in this article, and as IBEX 35 or the IBEX index thereafter.

insights/results for the dynamics of the heterogeneity of the Spanish market between January 1998 and June 2005 in terms of various return metrics. Section 4 provides results in terms of various risk metrics for the same period. Section 5 concludes and provides suggestions for future research.

2. METHODOLOGY AND DATA

2.1. Portfolio Opportunity Sets

The distinctive feature of our approach is not to consider a single benchmark index to describe a financial market, (and/or use it to measure investment performance), but instead to consider *all possible* portfolios. We call the set of all portfolios that are feasible under a specific investment mandate the *portfolio opportunity set*.[2] This set signifies a level playing field for any portfolio that aims for the same investment objectives and respects the same constraints as specified in the mandate. Next, we observe the dynamics of each feasible portfolio in this opportunity set and build frequency distribution(s) of performance values for performance measures of interest. Furthermore, we condense the information of the whole portfolio opportunity set (i.e. frequency distributions) into a compact and informative market description.

The advantage of our approach is that our market description can be made commensurate with any specific investment environment (such as an investment mandate, defined by goals and constraints). Thus, our "enhanced" description provides a comprehensive view on upside and downside potentials of the large cap segment (as represented by the IBEX 35 stocks) of the Spanish equity market. Furthermore, it provides valuable insights into relative performance of a portfolio, i.e. *how substantial* is the relative over- or under-performance of the portfolio with respect to the IBEX 35 index for a specific period. Our descriptions also help to evaluate whether a portfolio or a peer group (e.g. a specific group of mutual funds) exploit the available investment opportunities in a successful way.

The portfolio opportunity set used for the description and analysis in this article (henceforth denoted by $\mathcal{I}$) consists of all portfolios that can be composed from the contemporary 35 stocks that compose the IBEX 35 index, i.e. portfolios consisting of IBEX 35 stocks with weights subject to the following constraints:

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2   The portfolio opportunity set (POS) approach is different from the Wall Street Journal's dartboard portfolio approach, in which a random portfolio is constructed by "throwing darts" at the financial pages of the Wall Street Journal (see Moy (1994) and Adams & Cyree (2004), for example). In the WSJ approach, a single random portfolio is created by having a monkey choosing (by throwing darts) four different investment objects and investing 25% of the portfolio in each of these four stocks. The POS approach generates a large number of well-diversified portfolios.

$$\mathcal{I} = \{0 \le w_i \le 1.0 , i=1, 2, \dots , 35 \text{ such that } \sum_{i=1}^{35} w_i = 1\}. \tag{2.1}$$

Clearly, given these weight constraints, the number of portfolios in the opportunity set $\mathcal{I}$ is infinite. Therefore, we estimate the frequency distribution of performance values defined over the portfolio opportunity set from a reasonably-large sample.[3] The generation procedure is described in detail in Appendix A.

Generally, each of frequency distributions estimated by our procedure can be characterized by its first two standard moments: mean (as an indicator of the distribution location), and standard deviation (as an indicator of the distribution width). As we are interested in this article in investigation of cross-sectional diversity (and its dynamics) of portfolios for a given mandate, we concentrate on such standard deviation values of various frequency distributions for mandate $\mathcal{I}$ as defined by (2.1). Let us define in the next section the standard deviation based heterogeneity measure more precisely.

2.2. Measure of Market Heterogeneity

Informally, given investment mandate $\mathcal{I}$, we define *heterogeneity* with respect to a specific performance measure as the magnitude of differences between cross-sectional performances of portfolios, which are feasible for investment under $\mathcal{I}$. For example, speaking of heterogeneity of returns for a time period, we consider the magnitude of the differences between the cross-sectional portfolio returns over that period.

More formally, let $\mathcal{F}(\tau)$ denote the frequency density function on $\mathcal{I}$ with respect to a performance measure $\boldsymbol{\tau}$. Roughly speaking, function $\mathcal{F}(\tau)$ defines the proportion of portfolios from $\mathcal{I}$, which have a specific performance τ over period t.[4] Then, similarly to probabilistic distributions, we define the first moment of $\mathcal{F}(\tau)$, henceforth denoted by μ_τ, as:

$$\mu_\tau = \int_{\mathcal{I}} t \; \mathcal{F}(t) \; dt \tag{2.2}$$

consequently we define second moment of $\mathcal{F}(\tau)$, henceforth denoted by σ_τ, as:

$$\sigma_\tau^2 = \int_{\mathcal{I}} (t - m_\tau)^2 \; \mathcal{F}(t) \; dt \tag{2.3}$$

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3   Given a portfolio opportunity set, there are several ways to compute the required distribution(s). Simulation is one of them.

4   For the sake of understandability we do not write time superscript (t) when time interval is irrelevant or clear from context.

Furthermore, we define *heterogeneity* with respect to a given performance measure τ over a period t for a given mandate, denoted by $\mathcal{H}_\tau^{(t)}$, as:

$$\mathcal{H}_\tau^{(t)} = \sigma_\tau^{(t)} \tag{2.4}$$

That is, heterogeneity is the standard deviation of the corresponding frequency density function.

For some performance measures and mandates, an analytical expression for $\mathcal{F}(\cdot)$ does exist. In other cases, we have to numerically estimate the function $\mathcal{F}(\cdot)$ for our mandate $\mathcal{I}$. Therefore, we need to reformulate expressions (2.2) and (2.3) as sample-based estimates. The sample-based first moment can be calculated as:

$$\tilde{\iota}_t = \sum_{i=1}^{n} t_i \Big/ n \tag{2.5}$$

where n is the sample size and $\tau_1, \tau_2, \ldots \tau_n$ are performances of portfolios from sample. The sample second moment and, thus, sample-based heterogeneity is equal to:

$$\bar{\mathcal{H}}_t^2 = \bar{\sigma}_t^2 = \sum_{i=1}^{n} \left(t_i - \tilde{\iota}_t \right)^2 \Big/ (n-1) \tag{2.6}$$

We use the last expression to estimate the heterogeneity of $\mathcal{I}$ for various performance measures in our empirical analysis of the Spanish large-cap equity market in the next two sections.

2.3. Data

We used daily reports on the IBEX 35 as well as the "Composición histórica – Ibex 35" published by the Sociedad de Bolsas to comprise the list of stocks constituting the IBEX 35 index over the last ten years. The input data consists of monthly observations on these stocks from January 1996 through May 2005. We used closing prices at the last trading day of each month to compute discrete (percentage) returns.[5] The prices are corrected for all corporate actions (e.g. stock splits, capital increases, Euro introduction) but dividends.[6] The stock data were downloaded from the Thompson Datastream database and cross-checked against the data available through Bloomberg.

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5   We use last trading day monthly prices, so our evaluation horizon is one month. We do not use within-month averaged prices because this would generate various statistical biases in the return series, see for example Wilson, Jones & Lundstrum (2001) and Hallerbach (2003).
6   IBEX 35 is a price index.

For a few stocks such as Telefonica Moviles S.A. and Grupo Ferrovial S.A., only short historical time series exist before their inclusion in the index. Therefore, we performed the regression of such stocks on the IBEX 35 level and other index constituents in order to extend their time series to have 24 monthly return observations before the index inclusion.[7]

An important issue in our simulation procedure is how to handle changes in the index composition. Regular changes are carried out twice a year in July and January. However, the changes in the IBEX are quite irregular due to mergers, new admissions, deletions, etc., which should be reflected in the index shortly after their occurrence. After changes in the structure of the IBEX are decided upon by the Gestor de Índices IBEX S.A., they give a 4-6 weeks notice before implementation. Therefore our strategy is to hold the security deleted from the index until the start of the replacement month and then replace it by the new one. For example, on July 2, 2001, Picking Pack was exchanged in the index against Inditex. When we evaluate the performance of the feasible portfolios at the last trading day of June 2001, we have Picking Pack as one of the IBEX 35 stocks. For the evaluation month July 2001 the Picking Pack stock drops from the stock set and, thus, from the portfolios. Instead, Inditex will be used as a new stock in the IBEX 35 stock set to compose portfolios.

3. CROSS-SECTIONAL HETEROGENEITY OF STOCK RETURNS OF THE SPANISH EQUITY MARKET

As mentioned, we estimated $\bar{\mu}_{(\cdot)}$ (i.e. cross-sectional mean/average of portfolio performance values given $\mathcal{I}$) and the heterogeneity $\bar{\mathcal{H}}_{(\cdot)}$ given mandate $\mathcal{I}$. We did that for various performance metrics relating to portfolio return and portfolio risk. Let us look of different return measures first.

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7   We use a recursive regression approach to fill in missing observations. When there is one or more missing observations (for example when a stock is incorporated in the IBEX), we start with a window of 24 return observations on the stock i and the index I, which begins immediately after the missing return data point for the stock. Within this window, we use OLS regression to estimate the parameters α_t and β_t of the market model:

$$r_{i,t} = \alpha_t + \beta_t \, r_{I,t} + e_{i,t} \qquad t = 1,\ldots,24$$

Given these parameters and the available index return for the first observation t=0 before the start of the window, we construct the stock return, conditional on the index return:

$$r_{i,t} \big| r_{I,t} = \hat{\alpha}_{t} + \hat{\beta}_t \, r_{I,t} \qquad t=0$$

Finally, we randomly sample one residual term from the 24 residuals in the estimation window and add that to $r_{i,0} \big| r_{I,0}$. We substitute this return for the missing return $r_{i,0}$. We can repeat the procedure by moving the data window backwards in time.

Figure 1 presents the development of the trailing $\bar{\mu}_{r_{1M}}$ and $\bar{\mathcal{H}}_{r_{1M}}$ of one-month realized returns. (For the sake of clear visual appearance the values of $\bar{\mu}_{r_{1M}}$ are connected by a line.) Looking at the development of $\bar{\mu}_{r_{1M}}$ over time, from January 1998 to approximately the end of 2002, the changes in the month to month $\bar{\mu}_{r_{1M}}$ have more or less the same amplitude. From 2003 onwards this has changed: the amplitude of $\bar{1}_{r_{1M}}$ has decreased significantly. A similar observation can be made with respect to the development of the heterogeneity $\bar{\mathcal{H}}_{r_{1M}}$ of one-month returns as presented in the right graph of Figure 1. From begin of 2003 onwards the level of $\bar{\mathcal{H}}_{r_{1M}}$ has halved, from 1.5% to 0.8%. Also the month to month changes in $\bar{\mathcal{H}}_{r_{1M}}$ has decreased substantially. In plain language, speaking of one-month returns for portfolios composed from the IBEX 35 stocks, from the start of 2003 one would expect rather modest month-to-month changes in the average portfolio returns (about 4 till 10%). Simultaneously, the diversity in one-month returns across different portfolios has decreased significantly, and, thus, one could expect to deviate only marginally (about 0.8%) in return from the average value. So, in the period starting 2003, the IBEX 35 stocks cover a pretty homogeneous market.
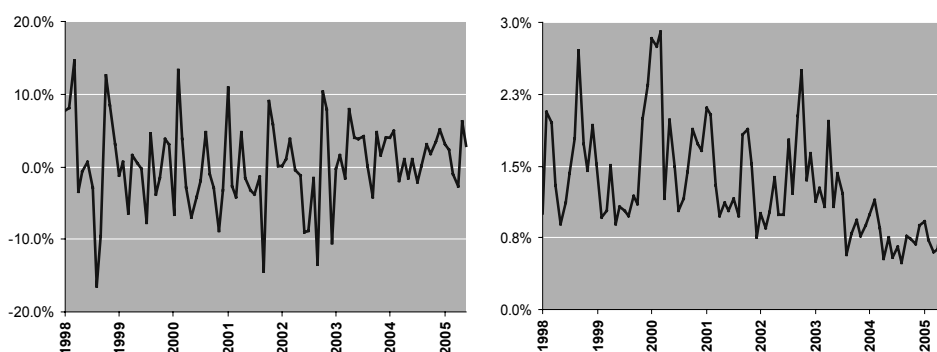

Figure 1: $\bar{\mu}_{r_{1M}}$ and $\bar{\mathcal{H}}_{r_{1M}}$ **of one-month realized returns.**

The left graph shows the trailing cross-sectional first moment $\bar{\mu}_{r_{1M}}$ of one-month realized returns for portfolios from $\mathcal{I}$ for the period from January 1998 through June 2005. The right graph shows the trailing heterogeneity $\bar{\mathcal{H}}_{r_{1M}}$ (i.e. cross-sectional standard deviation) of one-month realized returns for portfolios from $\mathcal{I}$ for the same period.

For the dataset at hand, it is hard to judge whether the levels of the metrics $\bar{\mu}_{r_{1M}}$ and $\bar{\mathcal{H}}_{r_{1M}}$ were high due to the bubble developing in the late nineties – start of the century, or the present low levels are rather extraordinary/unusual.

As second return metric, we have calculated the cross-sectional average of the individual monthly returns on the 35 stocks in the IBEX 35 as well as the cross-sectional standard deviation of this set of individual stock returns.[8] Figure 2 shows the results.[9] As expected, the '1998 and onwards' part of the left graph in Figure 2 coincides with the left graph in Figure 1. However, the cross-sectional standard deviation graphs in Figure 1 (right graph) and in Figure 2 (bottom graph) do not. The reason is that in the first case we consider all possible portfolios, and in the second case all single stocks. First, this shows that the cross-sectional average of individual stock returns is much more heterogeneous than the cross-sectional portfolio returns. And comparison of the heterogeneity changes over time shows that these changes have more or less the same sign but also that the changes in the heterogeneity of the cross-sectional average of individual stock returns are much bigger.
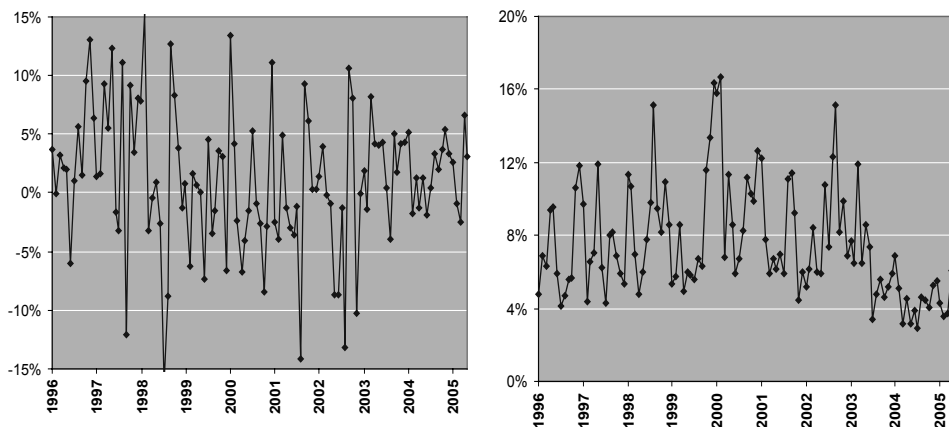

Figure 2: **Statistics of the cross-sectional returns of the IBEX stocks from 1996 through 2005.**

The left graph shows the cross-sectional averages of monthly returns of the IBEX stocks from 1996 through 2005. The right graph plots the cross-sectional standard deviations of these returns. In order to see the direction of change, we connected these statistics by a thin line.

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8   On the contrary to the portfolio opportunity set distributions, we can calculate these metrics from 1996 onwards.
9   Essentially, each dot in Figure 2 represents the return on the equally-weighted portfolio in the respective month, and the corresponding standard deviation specifies how much we can outperform/underperform by investing in only one stock of IBEX 35.

Summarizing the insights given by both figures, we can make the following observations toward the market heterogeneity $\overline{\mathcal{H}}_{r_{1M}}$ of one-month returns:

1.   The period from the end of 1996 to start of 2003 is characterized by an increased heterogeneity of portfolio returns. While till begin of 1996 as well as after 2002 portfolios deviate 0.8% from each other on average, in the time between these dates the deviation was twice as high and equal to 1.5% (again, on average);

2.   The month-to-month volatility of $\overline{\mathcal{H}}_{r_{1M}}$ has increased dramatically over the end of 1996 – end of 2003 period. In fact, the differences make up to 2% for some neighboring months. In the periods of a 'normal' market such as from start of 2003 till the end of the analyzed time horizon, the volatility in $\overline{\mathcal{H}}_{r_{1M}}$ is about 0.7% – 0.8%.

The third return metric considered is the 24-month average returns for portfolios from $\mathcal{I}$. Figure 3 shows the results of calculating $\overline{\mu}_{r_{AVG}}$ and $\overline{\mathcal{H}}_{r_{AVG}}$.

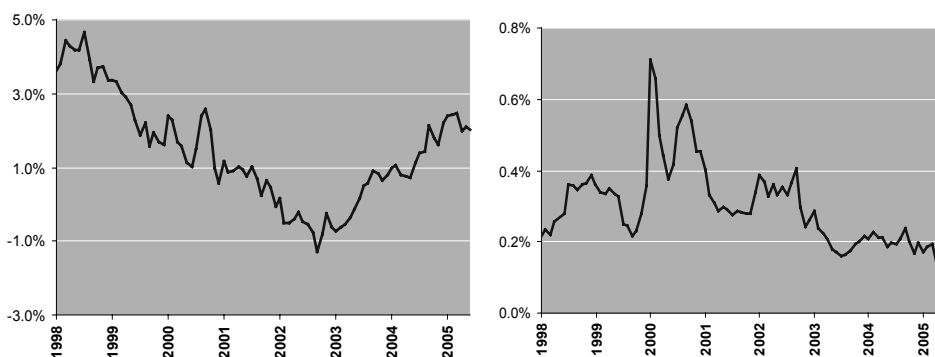**Figure 3:** $\overline{\mu}_{r_{AVG}}$ and $\overline{\mathcal{H}}_{r_{AVG}}$ **of the 24-month averages of monthly realized returns.**

The left graph shows the trailing cross-sectional first moment $\overline{\mu}_{r_{AVG}}$ of the 24-month averages of monthly realized returns for portfolios from $\mathcal{I}$ for the period from January 1998 through June 2005. The right graph shows the trailing heterogeneity $\overline{\mathcal{H}}_{r_{AVG}}$ (i.e. cross-sectional standard deviation) of the 24-month averages of monthly realized returns for portfolios from $\mathcal{I}$ for the same period.

Let us take a look at the dynamics of $\overline{\mu}_{r_{AVG}}$ as shown by the left graph of Figure 3. $\overline{\mu}_{r_{AVG}}$ reached the maximum of about 4.5% in the middle of 1998, afterwards it declined gradually till the second half of 2002, where $\overline{\mu}_{r_{AVG}}$ was even less than -1% once. Then it starts to recover. The final value of $\overline{\mu}_{r_{AVG}}$ in June 2005 was equal to 2.04%. Taking into account the clustering effect caused by averaging returns for the 24 months, the development of $\overline{\mu}_{r_{AVG}}$ fairly reflects the general market movement over time (cf. with the development of the IBEX 35 over time).

The development of $\bar{\mathcal{H}}_{r_{AVG}}$ is more surprising. In the periods of a 'normal' market such as at the begin of 1998 and from start of 2003 till the end of the analyzed time horizon, the heterogeneity of 24-month average returns has the level of about 0.2%. That is, considering more or less well-diversified portfolios composed from the IBEX 35 stocks, the window of return opportunities is pretty narrow – about 4.8% per annum – under/in a 'normal' market. Beyond the 'normal' time periods the graph shows three periods where we observe a fully different picture. The first is from the middle of 1998 to the middle of 1999. In this period $\bar{\mathcal{H}}_{r_{AVG}}$ has value of about 0.4%, which corresponds to 9.6% per annum. That is, there was a plenty of opportunities to overperform (as well as underperform) significantly in that period. A similar 'bump' we have from the end of 2001 to the end of 2003. An interesting point is that whereas the first abnormality corresponds to a strong bull market, the second one corresponds to a strong bear market. Furthermore, the highest heterogeneity of 0.6%–0.7% (i.e. 14.4 – 16.8% p.a.) is observed around year 2000 (The market went sharply up and then down in that period). And values of $\bar{\mathcal{H}}_{r_{AVG}}$ in that period suggest that enormous diversity in performances was present on the market; also the period is characterized by a rapid change in heterogeneity from month to month.

In general, as we considering consequences and/or applications of market heterogeneity, the heterogeneity metrics $\bar{\mathcal{H}}_{(\cdot)}$ help to quantify the effect of portfolio flexibility on the performance. For example, considering $\bar{\mathcal{H}}_{r_{1M}}$ through 2004–2005, one would expect that the different portfolio compositions attribute up to 0.8% of difference in one-month return. Another consequence is that $\bar{\mathcal{H}}_{(\cdot)}$ allows evaluating the relative performance of a manager or a portfolio in a more precise way. Depending on a specific $\bar{\mathcal{H}}_{(\cdot)}$ value, an outperformance – let say of 0.25% – by a portfolio over a benchmark may be either substantial or not. For example, considering $\bar{\mathcal{H}}_{r_{1M}}$ through 2004–2005, one would value such an outperformance as a very good one, while in 1999 it is just adequate. Other applications can also be derived.

It should be noted that the heterogeneity metrics $\bar{\mathcal{H}}_{(\cdot)}$ can be considered with respect to any performance measure of interest. Using, for example, the Sharpe ratio (SR) for evaluating performance of a portfolio, the market heterogeneity $\bar{\mathcal{H}}_{SR}$ will surely provide valuable insights, which can be used for evaluation etc. However, as in this article we are interested to provide various general insights on the Spanish market, we do not discuss the applications of $\bar{\mathcal{H}}_{(\cdot)}$. For a detailed overview of performance evaluation using portfolio opportunity sets and diverse $\mathcal{H}_{(\cdot)}$, we refer to Pouchkarev (2005).

The existence of rapid changes in $\bar{\mu}_{(\cdot)}$ and $\bar{\mathcal{H}}_{(\cdot)}$ for different return measures suggest that risk pictures should be affected as well. The dynamics of $\bar{\mu}_{(\cdot)}$ and $\bar{\mathcal{H}}_{(\cdot)}$ with respect to different risk measures will be presented in the next section.

4. CROSS-SECTIONAL HETEROGENEITY OF STOCK RISKS OF THE SPANISH EQUITY MARKET

Figure 4 presents the trailing $\bar{\mu}_{s_P}$ and $\bar{\mathcal{H}}_{s_P}$ of the 24-month standard deviations (of monthly returns) for the period from January 1998 through June 2005. (In order to avoid confusions we denote standard deviation of a portfolio P from $\mathcal{I}$ by s_P and standard deviation of the distribution of s_P's for all portfolios from $\mathcal{I}$ by $\bar{s}_{s_P}^{(t)}$ henceforth.)

As we would expect, the general level of risk, as represented by $\bar{\mu}_{s_P}$, has increased significantly during the boom-and-burst period of 1998 – 2003 and reached about 7.5% per month (we can also observe a sharp decrease in the level of $\bar{\mu}_{s_P}$ in the middle of 2004). Afterwards, we observe a rather modest 3% level of risk in monthly returns for the average portfolio from $\mathcal{I}$. This development is in line with our $\bar{\mu}_{r_{1M}}$ graph in Figure 3.
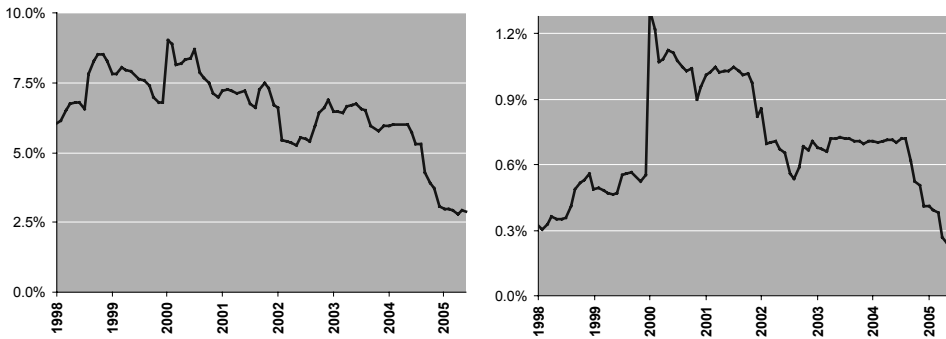

Figure 4: $\bar{\mu}_{s_P}$ and $\bar{\mathcal{H}}_{s_P}$ of the 24-month standard deviations.

The left graph shows the trailing cross-sectional first moment $\bar{\mu}_{s_P}$ of the 24-month standard deviations (of monthly returns) for portfolios from $\mathcal{I}$ for the period from January 1998 through June 2005. The right graph shows the trailing heterogeneity $\bar{\mathcal{H}}_{s_P}$ (i.e. cross-sectional standard deviation) of the 24-month standard deviations for portfolios from $\mathcal{I}$ for the same period.

A number of interesting new insights is brought by the $\bar{\mathcal{H}}_{s_P}$ graph on the right in Figure 4. From the start of 1998 until the second half of 1999 we see the steady increase from 0.3% to 0.6% in the risk heterogeneity for portfolios from $\mathcal{I}$. Clearly, in a booming market one would expect that there exists more variety in portfolios

than in a 'normal' market: Some portfolios would perform stronger than usually, some would provide 'standard' performance. At the end of 1999 – start of 2000 we observe an impetuous increase of $\overline{\mathcal{H}}_{s_P}$: during a few months it jumped from 0.6% to 1.2%. This peak is caused by extreme discrepancies between stock returns between November 1999 and May 2000 (cf. the right graph in Figure 1). From the end of 2002 until mid 2004, we see a 'plateau' of a still high heterogeneity of about 7.5%, which is then followed by a sharp decrease of the heterogeneity in the risk of the IBEX portfolios.

Overall, using the $\overline{\mu}_{s_P}$ and $\overline{\mathcal{H}}_{s_P}$ measures, we can easily and unambiguously quantify the general level of risk (as represented by $\overline{\mu}_{s_P}$) as well as the standardized interval for possible risk values for portfolios in $\mathcal{I}$. For example, running a portfolio that satisfy the constraints of $\mathcal{I}$, one would experience 2.6% – 3.2% risk in monthly portfolio returns through the first half of 2005. Thus, the $\overline{\mathcal{H}}_{s_P}$ measure signals the contemporary diversity level of portfolios for a given investment mandate ($\mathcal{I}$ in our case). $\overline{\mathcal{H}}_{s_P}$ also quantifies the potential for the risk decrease (the attainable lower risk bound) available on the market, which could be realized via, for example, additional diversification, security selection and so on.

Both the average portfolio risk and the heterogeneity of portfolio risk within the portfolio opportunity set are a function of the covariances of the stocks in the portfolio opportunity set. The changes in the level of $\overline{\mathcal{H}}_{s_P}$ over time reflect the changes in the covariance structure of the underlying investment opportunity set – 35 stocks composing the IBEX index in our case. And as the $\overline{\mathcal{H}}_{s_P}$ graph in Figure 4 suggests, the heterogeneity level was almost three times as high in some periods than in other periods. To get more insight into the development of the covariance structure, we calculated at the end of each month of the sample period the covariance matrix and the corresponding correlation matrix using the preceding 24 monthly return observations of the IBEX-35 stocks. The two graphs in Figure 5 show the trailing averages (on the left) and standard deviations (on the right) of the covariance matrix entries. The increase of the average covariance values between 1998 – 2004 coincides with the observation that the common risk level is increased over that time (the level of the variance/covariance entries have risen substantially). And the rise in the standard deviation of the covariance terms represents the higher level of heterogeneity of the stock risks. In 2001 both the averages (with a brief 'revival' at the end of 2002) and the standard deviations of the covariance entries start to decrease until 2005, the end of our data series.
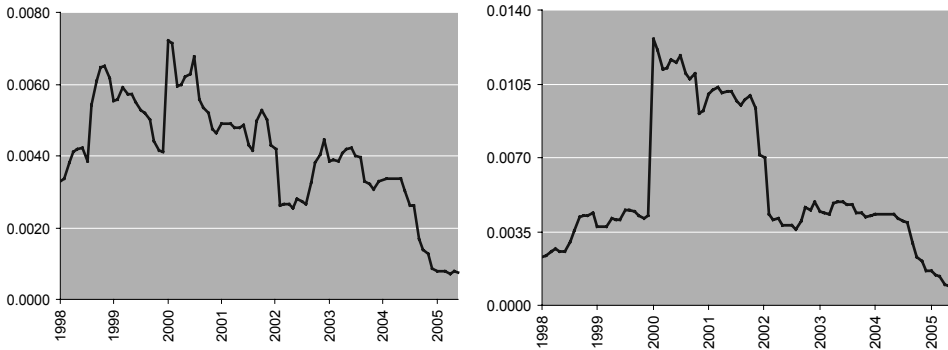
Figure 5: **Statistics of the covariance matrix structure for the IBEX stocks from 1998 through 2005.**

The graphs show the cross-sectional averages (left) and standard deviations (right) of the 24-month trailing covariance matrix entries.

The two bottom graphs in Figure 6 show the corresponding averages and standard deviations of the correlation matrix entries.
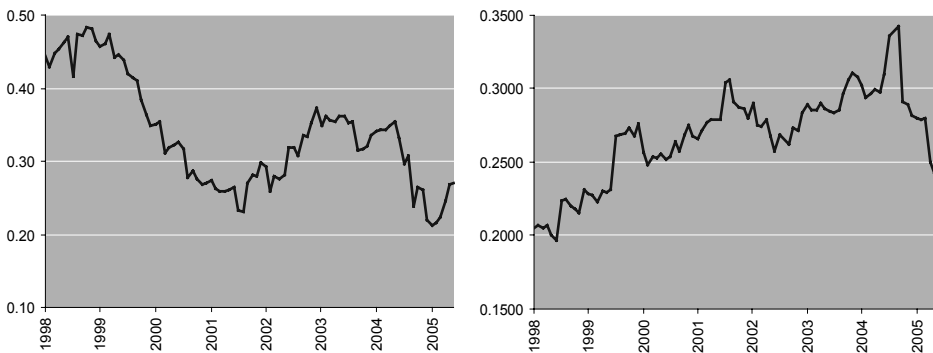

Figure 6: **Statistics of the correlation matrix structure for the IBEX stocks from 1998 through 2005.**

The graphs show the cross-sectional averages (left) and standard deviations (right) of the 24-month trailing correlation matrix entries.

The left graph in Figure 6 suggests that stocks are more loosely correlated from the end of 1999 onwards. However, we observe some "recovery" in the correlations between 2002 and 2004 as well as in the last year of our sample period. The right graph of Figure 6 clearly shows that the heterogeneity of correlation matrix entries has strongly increased over time. That is, the stocks move lees in line. Also we can identify the same three peaks (although very smoothed) as on the graphs in Figure

4. Again, Figure 5, Figure 6, and Figure 4 present similar information from different perspectives.

It should be noted that beyond the $\overline{\mathcal{H}}_{s_P}$ measure, we also analyzed the dynamics of heterogeneity for other commonly-used risk metrics such as downside semi-variance, mean-absolute deviation (equally and exponentially-smoothed variants), Gini coefficient etc. For space considerations as well as they show a similar picture, we do not present these graphs in the paper but made them available online.

5. CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

In this article, we introduce the concept of market *heterogeneity*, which is a measure of the cross-sectional diversity of the underlying (equity) market. This diversity can relate to the returns of the portfolios implied by the opportunity set, to the risk or any other performance measure defined for these portfolios. The heterogeneity (of the portfolio opportunity set) is a new concept, which produces additional insights into the nature of the portfolio opportunity set, market, which is the domain for the portfolio concerned.

In particular, we analyze the development of the heterogeneity of the Spanish stock market (as represented by the IBEX 35 index), both in terms of returns and in terms of risk. In terms of returns we consider the heterogeneity of one-month returns, of single IBEX stocks and of 24-month average returns for the IBEX 35 portfolio opportunity set. In terms of risks we analyze the heterogeneity of portfolio standard deviations. In addition, we calculate the variance-covariance matrix of the IBEX 35 stocks on a trailing basis and analyze the dynamics of its structure over time. The results provide new interesting insights on the cross-sectional development of the Spanish market over time. Also they are potentially very interesting for relative performance evaluation. For example, considering tracking error volatility (TEV) constrained investments, our heterogeneity measure(s) can help dynamically budget risk.

We showed primary the construction and dynamics of the market heterogeneity measure(s). This suggests an interesting route for further research: to use the portfolio heterogeneity measure(s) to evaluate the relative performance of different managers or strategies using, for example, the normalized versions of excess returns and information ratios.

Acknowledgments

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APPENDIX A GENERATION PROCEDURE FOR PORTFOLIO OPPORTUNITY SET SAMPLE

We use the following procedure to generate the portfolio sample for the IBEX investment mandate defined by (2.4):

I.   In each simulation step we sample ten million feasible random portfolio weight vectors for stocks of the IBEX 35. We use the following algorithm adapted from Devroye (1986) to generate random portfolios:

1. Generate U_1, U_2, …, U_{34} as 34 independent [0, 1] uniformly distributed random variables (i.e. $U_i \sim \mathcal{U}(0,1)$, $i = 1, 2, …, 34$);
2. Sort U_i in ascending order, i.e. define the order statistics $U_{(1)}$, $U_{(2)}$, …, $U_{(34)}$;
3. Create a new random portfolio with asset weights (w_1, w_2, …, w_{35}) where $w_i = U_{(i)} - U_{(i-1)}$, $1 \leq i \leq 35$, $U_{(0)} = 0$ and $U_{(35)} = 1$;
4. If the sample is big enough, then stop. Otherwise return to step 1.

Each sampled weight vector defines a feasible portfolio and is an alternative for investing in the Spanish large cap segment. The sampled portfolios are uniformly distributed over the IBEX 35 portfolio opportunity set;

II.   For these sampled portfolios, as well as for the actual IBEX 35 index and the equally weighted benchmark, we calculate different portfolio return characteristics: realized discrete (percentage) returns over different periods, the arithmetic average rate of returns, standard deviations, and semi-standard deviations.[10] Except for realized returns, these statistics are estimated using 24 monthly observations prior to the actual evaluation period (For example, for evaluating the market during May 2003, the stock returns from May 2001 through April 2003 are used);

We estimate the frequency distributions of the selected performance measures over the whole portfolio opportunity set;

III.   The time window is shifted one month forward and the next simulation is carried out.

Thus, steps I-III provides a "snapshot" of existing opportunities available on the Spanish blue-chip equity market over a certain time period. The information content of these steps is the location and form of the computed distributions of performance values. Step IV provides a picture of the development of the market dynamics over time by showing the development of the dispersion of these distributions.

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10 The number of characteristics can be extended at will, without any problem. Examples are mean absolute deviation, the Sharpe ratio, the Treynor ratio and many others.